The Measure

Artist's Proof 25

Born Rule

The Born rule as the sound the axioms make on Hilbert space

Status and Dependency

This paper derives the probability structure of actualisation from the axioms. It closes two gaps identified as upstream caps in AP23 (The Single Record).

Gap 1: AP23's Lemma 1 relies on the claim that reduced density matrices are not record targets — that actualisation targets pure states only.

This was stated as a commitment in AP23's notation section but not formally derived from the axioms.

Gap 2: AP23's Proposition 2 (Bell Prediction) and the CHSH quantitative payload use the Born rule. The Born rule was established in AP07 but not derived from {S, B, R, C}.

It was imported as a feature of the complex Hilbert space structure.

This paper derives both. Proposition 1 shows that Axiom R's requirement for definite records forces actualisation to target pure states. Proposition 2 shows that Axiom R and Axiom B jointly imply non-contextual outcome probabilities.

Lemma 1 establishes orthogonal additivity.

Theorem 1 (The Measure) shows that the unique probability measure consistent with these requirements on a complex Hilbert space of dimension ≥ 3 is the Born rule: $P(a) = \mathrm{Tr}(|\psi\rangle\langle\psi| \cdot P_a)$.

For rank-1 projectors, this gives $|\langle a|\psi\rangle|^2$. The Born rule is not imported. It is derived.

The dependency chain: Axiom S (distinction) → Axiom B (single break) → Axiom R (definite, irreversible record) → AP07 (complex Hilbert space) → AP10 (3 spatial dimensions; physical wavefunctions live in $L^2(R^3)$, which is separable and infinite-dimensional, so the full physical Hilbert space satisfies $\dim(H) \geq 3$) → this paper (pure-state targets, non-contextuality, orthogonal additivity, Born rule).

Epistemic status per section. §1 (The Two Gaps): historical. §2 (What Definite Means): derived. §3 (Record Targets Are Pure States): derived — Proposition 1. §4

(Non-Contextuality): derived — Proposition 2. §5 (Orthogonal Additivity): derived — Lemma 1. §6 (The Born Rule): derived — Theorem 1. §7 (What This Closes): derived — retroactive strengthening. §8 (The Deeper Point): synthesis — non-load-bearing.

Notation

$|\psi\rangle$ — a pure state (ray) in Hilbert space H. A single, definite quantum state. An extremal point of the convex set of density operators on H.

$\rho = \sum p_i |\psi_i\rangle\langle\psi_i|$ — a mixed state (density operator). A non-extremal element of the convex set. Admits multiple decompositions into pure states.

P_a — a projector onto the outcome subspace for measurement result a. In the non-degenerate case, $P_a = |a\rangle\langle a|$ (rank 1).

In the degenerate case, P_a has rank > 1, projecting onto a subspace of dimension > 1.

$Tr(\rho \cdot P_a)$ — the trace of ρ composed with P_a. The Born rule in its general form.

dim(H) — the dimension of the Hilbert space H.

Frame function — a function μ assigning a non-negative real number to each projection operator in H, such that $\sum \mu(P_i) = 1$ for every resolution of the identity (every complete set of mutually orthogonal projectors $\{P_i\}$ with $\sum P_i = I$).

A frame function is defined on projectors alone — not on (projector, context) pairs. This is Gleason non-contextuality.

Gleason non-contextuality — the probability $\mu(P_a)$ assigned to projector P_a depends only on P_a itself, not on which resolution of the identity (which PVM) P_a belongs to.

This is a property of the probability assignment, not of pre-existing value assignments. It is distinct from Kochen-Specker contextuality, which concerns the impossibility of non-contextual definite-value assignments to all observables simultaneously.

The axioms deny pre-existing values (Axiom R), so KS contextuality is expected; Gleason non-contextuality of outcome probabilities (Proposition 2) is a separate, compatible claim.

Kill Switches

KS-55 (Gleason non-contextuality): LIVE — EMPIRICAL. Structurally secure.

KS-56 (Non-Born statistics): LIVE — EMPIRICAL. All experiments confirm Born rule.

KS-57 (Mixed-state actualisation): LIVE — EMPIRICAL. No known process actualises mixed states directly.

Here is how to destroy this paper. Find a physical system whose measurement statistics violate the Born rule.

Or demonstrate contextual outcome probabilities — where the probability of result a depends on which other measurement is performed alongside it.

Or show that a mixed state can be directly actualised — that the universe can write a record that is inherently ambiguous about which pure state obtains. Any of these kills the derivation.

§1 — The Two Gaps

You have rolled dice your entire life. You have never asked why the probability of rolling a six is one-sixth. The die has six faces. The faces are symmetric. The probability follows.

But quantum mechanics is not a die. The probabilities are the squares of amplitudes. Not the amplitudes themselves. Not the cubes. The squares. Why?

The axioms' account of quantum mechanics rests on two claims about actualisation. First: actualisation targets pure states. Second: the probability of each outcome is given by the Born rule.

Both claims are used throughout the corpus. Neither has been derived from the axioms.

Gap 1 (Record targets). AP23 (The Single Record) defines a record target as a pure state and shows that reduced density matrices of entangled systems are not record targets.

This is used in Lemma 1 to prove that local interaction with an entangled subsystem actualises the global state. But the definition is stated as a commitment, not derived from Axiom R.

Why must record targets be pure states? What in the axioms forbids actualisation of a mixed state?

Gap 2 (Born rule). AP07 establishes the complex Hilbert space and the Born rule as the probability rule for measurement outcomes.

The CHSH computation in AP23 (§6.0.1) uses the Born rule to derive $S = 2\sqrt{2}$. But the Born rule itself was introduced as a feature of the Hilbert space structure, not derived from {S, B, R, C}.

If the axioms claim to derive physics from four axioms, the probability measure that governs actualisation should follow from those axioms, not be imported alongside them.

This paper closes both gaps. The derivation chain is: Axiom R (definite records) → pure-state targets → non-contextuality → orthogonal additivity → Gleason's theorem → Born rule. Every physical assumption comes from the axioms.

The only external input is Gleason's theorem itself, which is a mathematical theorem about Hilbert spaces — not a physical postulate.

§2 — What "Definite" Means

Axiom R says: the break writes a definite, irreversible record. The record is a fact about what happened. It answers the question completely. There is no residual ambiguity about what the record says.

What does "definite" mean in the language of Hilbert space?

The space of quantum states has a convex structure. Any probabilistic mixture of two states is also a state.

Pure states are the extremal points of this convex set: they cannot be written as a non-trivial mixture of other states.

Mixed states are non-extremal: they sit inside the convex body and can be decomposed into mixtures of pure states in multiple distinct ways.

A definite record corresponds to an extremal post-actualisation state — one with no residual convex ambiguity. If the post-actualisation state is extremal (pure), there is exactly one state consistent with the record.

If the post-actualisation state is non-extremal (mixed), the same density operator ρ can be decomposed as $\rho = \sum p_i |\psi_i\rangle\langle\psi_i| = \sum q_j |\varphi_j\rangle\langle\varphi_j|$ with $\{|\psi_i\rangle\} \neq \{|\varphi_j\rangle\}$.

The record "ρ was actualised" is compatible with multiple distinct underlying realities.

Even if the record attempted to specify a particular decomposition, there is no fact of the matter about which decomposition is correct — the same ρ is compatible with infinitely many choices.

A definite record must correspond to a single physical description, not a class of descriptions. Definiteness requires extremality. In Hilbert space, extremal states are pure states.

You have signed a contract. The signature is definite — it is yours, not a superposition of signatures. It admits no ambiguity about who signed.

That is what Axiom R demands of every record: one answer, no residual uncertainty, no alternative decomposition. A mixed state is a contract with multiple possible signatories. The universe does not write ambiguous contracts.

§3 — Record Targets Are Pure States

Proposition 1 (Record Target). Let H be the Hilbert space of the pre-state (AP07). The states eligible for actualisation under Axiom R — record targets — are pure states (extremal points of the state space).

Mixed states (non-extremal points) are not record targets.

Proof. Axiom R requires that actualisation writes a definite record. By §2, a definite record corresponds to an extremal post-actualisation state: one that admits no non-trivial convex decomposition. Consider a candidate record target ρ.

If ρ is a mixed state (non-extremal, rank > 1), it admits multiple distinct decompositions into pure states. The record "ρ was actualised" does not uniquely specify the physical state.

It is compatible with infinitely many distinct decompositions, none of which is privileged. This violates Axiom R's requirement for definiteness. If ρ is a pure state (extremal, rank 1), then $\rho = |\psi\rangle\langle\psi|$.

It admits no non-trivial decomposition. The state is $|\psi\rangle$ and nothing else. The record is definite. Therefore: record targets are pure states. Mixed states are not eligible for actualisation. ■

Note on improper mixtures. The reduced density matrix of an entangled subsystem ($\rho\_A = Tr\_B(|AB\rangle\langle AB|)$) is an improper mixture: it arises from tracing out the entangled partner, not from classical ignorance.

Proposition 1 excludes improper mixtures from actualisation for the same reason as proper mixtures — they are non-extremal — but the improper case has an additional reason: the reduced state is not an independent state at all.

It is a marginal of the global entangled state. It has no standalone existence.

This is consistent with AP23's Lemma 1: local interaction with an entangled subsystem actualises the global pure state $|AB\rangle$, not the reduced state ρ_A.

You have just watched Axiom R — one requirement, definiteness — kill mixed-state actualisation. The universe cannot write a record that says “maybe this, maybe that.” It writes a record that says “this.”

Pure states are the only states that say “this” without ambiguity.

§4 — Non-Contextuality

Actualisation writes a definite record (Axiom R). The record answers a specific question: "what is the outcome of this measurement on this state?"

The next question is: can the probability of the outcome depend on what other measurements are performed simultaneously?

In quantum mechanics, two observables A and B are compatible (they can be measured simultaneously) if and only if they commute: $[A, B] = 0$. Contextuality would mean that the probability of getting outcome a when measuring A depends on whether A was measured alone, or alongside compatible observable B, or alongside compatible observable C.

The same measurement, the same state, but different probabilities depending on context.

Proposition 2 (Non-Contextuality). Let $|\psi\rangle$ be a pure state and P_a a projector corresponding to measurement outcome a. The probability $\mu(P_a)$ assigned by actualisation depends only on $|\psi\rangle$ and P_a.

It does not depend on which resolution of the identity P_a belongs to. Actualisation is Gleason non-contextual.

Proof. Axiom B provides one break. Axiom R writes one record for that break. The record registers the outcome of the actualisation event.

Identify what determines the record by examining what the axioms provide to the actualisation process.

Input 1: the state $|\psi\rangle$ (the pre-state being actualised). Input 2: the projector P_a (the outcome registered by the break). These two inputs are the complete specification of the actualisation event under the axioms.

Axiom S provides the distinction (the outcome is distinguished from alternatives). Axiom B provides the single break. Axiom R writes the record for the state and the outcome.

Now suppose the probability of outcome a depended on context — on which resolution of the identity $\{P_1, \ldots, P_n\}$ the projector P_a belongs to.

This context would constitute a third input to the actualisation process: the record would depend on $(|\psi\rangle, P_a, \{P_1, \ldots, P_n\})$. But the axioms provide no mechanism for this third input.

Axiom B says: one break. The break is the actualisation of the state by the projector. It does not include a specification of what other measurements could have been performed.

The break has no "compatible observable" register. Axiom R says: the record registers what happened — outcome a — not what could have happened.

The counterfactual alternatives (the other projectors in the resolution) are properties of the measurement apparatus on the manifold. They determine which outcomes are possible but are not inscribed in the record.

The record contains the state and the outcome. Context is not recorded because context is not an event.

Therefore: $\mu(P_a) = f(|\psi\rangle, P_a)$ for some function f that depends only on the state and the projector.

Equivalently: the probability assigned to P_a does not depend on which resolution of the identity P_a belongs to. Actualisation is Gleason non-contextual. ■

§5 — Orthogonal Additivity

Before invoking Gleason's theorem, one further condition must be established: orthogonal additivity. This is the property that makes μ a measure on the projection lattice, not merely a function on individual projectors.

Lemma 1 (Orthogonal Additivity). Let $\{P_1, \ldots, P_k\}$ be mutually orthogonal projectors and let $P = P_1 + \ldots + P_k$ be the projector onto their joint subspace (the coarse-grained outcome "one of outcomes 1 through k occurred").

Then $\mu(P) = \mu(P_1) + \ldots + \mu(P_k)$.

Proof. The coarse-grained outcome P corresponds to not distinguishing among the mutually exclusive fine-grained outcomes $P_1, \ldots, P_k$. By Axiom S, each fine-grained outcome is a distinct possibility. By Axiom B, one break occurs.

By Axiom R, the break produces exactly one outcome. The fine-grained outcomes are mutually exclusive (orthogonal projectors) and the coarse-grained outcome P is their disjunction.

The probability of the disjunction of mutually exclusive events is the sum of their individual probabilities — this is the defining property of a probability measure, which follows from Axiom B (one break = one outcome from the set of possibilities) and Axiom R (the record is definite = exactly one outcome is actualised).

Therefore $\mu(P) = \sum \mu(P_i)$. ■

Note: Lemma 1 is what makes μ a frame function in the sense required by Gleason's theorem.

A frame function is not merely a function on projectors that sums to 1 over resolutions of the identity — it respects the lattice structure of projections via orthogonal additivity.

Propositions 1 and 2 establish that μ is well-defined on projectors and independent of context. Lemma 1 establishes that μ is additive over orthogonal projectors. Together, these conditions are exactly the hypotheses of Gleason's theorem.

§6 — The Born Rule

Four ingredients, all derived from or established by the axioms:

- The pre-state is a complex, separable Hilbert space H with dim(H) ≥ 3 (AP07 + AP10). Separability (countable orthonormal basis) is satisfied by all physical Hilbert spaces: $L^2(R^3)$ is separable by construction.

- Actualisation targets pure states (Proposition 1, from Axiom R).

- The probability of each outcome is a Gleason non-contextual function of the state and the projector (Proposition 2, from Axiom R + Axiom B).

- The probability measure is orthogonally additive and normalised (Lemma 1, from Axioms S + B + R).

Conditions (2)–(4) together define a frame function on H: a non-negative, orthogonally additive function μ on projection operators, defined on projectors alone (not on context), with ∑ μ(P_i) = 1 for every resolution of the identity.

This is exactly the object to which Gleason's theorem applies.

Gleason's Theorem (A. M. Gleason, 1957).

Let H be a separable Hilbert space over C with dim(H) ≥ 3. Let μ be a frame function on H: a non-negative, orthogonally additive function on projection operators such that for any complete set of mutually orthogonal projectors {P_i} with ∑ P_i = I, we have ∑ μ(P_i) = 1. Then there exists a unique density operator ρ such that μ(P) = Tr(ρ · P) for all projectors P.

Gleason's theorem is a mathematical result about probability measures on Hilbert spaces. It is not a physical postulate.

It is in the same category as the spectral theorem or Noether's theorem: a piece of mathematics that the axioms use to derive consequences.

The axioms provide the physical hypotheses (complex Hilbert space, definite records, non-contextuality, orthogonal additivity, normalisation). Gleason provides the mathematical conclusion.

Theorem 1 (The Measure). The unique probability measure on outcomes of actualisation, consistent with the axioms, is the Born rule: $\mu(P) = \mathrm{Tr}(|\psi\rangle\langle\psi| \cdot P)$.

For rank-1 projectors $P\_a = |a\rangle\langle a|$, this gives $\mu(P\_a) = |\langle a|\psi\rangle|^2$.

Proof. The pre-state is a complex, separable Hilbert space H with $\dim(H) \geq 3$ (AP07, AP10). By Proposition 1, actualisation targets pure states $|\psi\rangle$.

By Proposition 2, the probability assigned to projector P is Gleason non-contextual: it depends only on $|\psi\rangle$ and P, not on which resolution of the identity P belongs to.

By Lemma 1, the probability measure is orthogonally additive. By normalisation (Axioms S + B), probabilities for a complete set of outcomes sum to 1. These conditions define a frame function on H.

By Gleason's theorem, the unique frame function on a complex, separable Hilbert space of dimension ≥ 3 is $\mu(P) = \mathrm{Tr}(\rho \cdot P)$.

Since the state being actualised is a pure state $|\psi\rangle$ (Proposition 1), $\rho = |\psi\rangle\langle\psi|$ and therefore $\mu(P) = \mathrm{Tr}(|\psi\rangle\langle\psi| \cdot P)$. For rank-1 projectors, this is $|\langle a|\psi\rangle|^2$. This is the Born rule. ■

You have just watched four axioms force the most mysterious equation in quantum mechanics. Not choose it. Not postulate it. Force it. There is no other measure.

Gleason's theorem does not say the Born rule is a good choice. It says the Born rule is the only choice. The probability of quantum outcomes was never a postulate.

It was a consequence — waiting in the structure of Hilbert space for the axioms to land on it.

Intuition: why the Born rule is forced. The elementary core of Gleason's result (restricted to rank-1 projectors) can be sketched without the full Gleason machinery.

Non-contextuality and normalisation together imply that the weight function f (assigning probabilities to rays) is additive: f(s + t) = f(s) + f(t) for overlaps of orthogonal subspaces. Non-negativity implies monotonicity.

A monotone additive function on [0,1] with f(0) = 0 and f(1) = 1 is the identity: f(t) = t. Therefore $\mu(P\_a) = |\langle a|\psi\rangle|^2$ — the Born rule. This is the Cauchy functional equation route.

Gleason's theorem generalises this to arbitrary projectors (not just rank-1) and to the full lattice structure, which is why the paper uses Gleason rather than the Cauchy argument directly.

Example: Spin measurement in full Hilbert space. Consider an electron with spin. The full Hilbert space is $H = L^2(R^3) \otimes C^2$, where $L^2(R^3)$ encodes spatial degrees of freedom and C^2 encodes spin.

This space is infinite-dimensional (dim ≥ 3 satisfied). It is separable ($L^2(R^3)$ has a countable orthonormal basis).

Prepare the spin in state $|\psi\rangle = \alpha|\uparrow\rangle + \beta|\downarrow\rangle$ with $|\alpha|^2 + |\beta|^2 = 1$. Measure spin along z. The projectors are $P\_\uparrow = |\uparrow\rangle\langle\uparrow|$ and $P\_\downarrow = |\downarrow\rangle\langle\downarrow|$ (rank-1).

By Theorem 1: $\mu(P\_\uparrow) = |\langle\uparrow|\psi\rangle|^2 = |\alpha|^2$. $\mu(P\_\downarrow) = |\langle\downarrow|\psi\rangle|^2 = |\beta|^2$. These are the Born rule probabilities. They sum to 1.

The spin-only Hilbert space C^2 has dimension 2, where Gleason's theorem does not apply directly.

But Theorem 1 is proved on the full physical Hilbert space $H = L^2(R^3) \otimes C^2$, where dim ≥ 3 is satisfied.

The Born rule on C^2 is inherited by restriction: the probability measure on the full space, restricted to the spin subspace, gives the same Born rule probabilities.

The dim ≥ 3 requirement applies to the full pre-state, not to effective subspaces used in calculations.

Uniqueness is crucial. Gleason's theorem does not merely say the Born rule is consistent with the axiom-derived conditions — it says the Born rule is the only measure that satisfies them. There is no alternative.

Any non-contextual, orthogonally additive, normalised probability assignment on a complex, separable Hilbert space of dimension ≥ 3 is the Born rule. The probability structure of actualisation is not a choice. It is forced.

§7 — What This Closes

The Born rule is no longer imported. It is derived. This retroactively strengthens every paper in the corpus that uses the Born rule.

AP07 (Complex Hilbert Space). AP07 established the complex Hilbert space and introduced the Born rule as a feature of that structure.

Theorem 1 replaces that introduction with a derivation: the Born rule is the unique probability measure forced by Axiom R on a complex Hilbert space of dimension ≥ 3. The Born rule is no longer a co-traveller of the Hilbert space.

It is a consequence of the axioms acting on that space.

AP23 (The Single Record). AP23's two upstream caps are closed. Cap 1: Proposition 1 derives the pure-state record target requirement from Axiom R's definiteness condition via convex geometry, replacing AP23's definitional commitment.

AP23's Lemma 1 (Local Interaction, Global Actualisation) now rests on a derivation, not a definition.

Cap 2: The CHSH computation (AP23 §6.0.1) uses the Born rule to derive $S = 2\sqrt{2}$. Theorem 1 derives the Born rule from {S, B, R, C}.

Therefore $S = 2\sqrt{2}$ is derived from the axioms. The Bell prediction is now fully axiomatic.

The corpus. Every paper that uses probability — AP09 (measurement as actualisation), AP11 (spin statistics), AP15 (charge quantisation), AP16 (electroweak unification), AP19 (SU(3) colour), AP21 (cosmic web) — is retroactively strengthened.

The probability structure was always the Born rule; now it is the Born rule because it is the only possibility.

§8 — The Deeper Point

Synthesis note: the following is non-load-bearing language. It carries no epistemic weight beyond the claims established above.

The Born rule is one of the most mysterious features of quantum mechanics. It tells us that the probability of finding a system in a particular state is the square of the amplitude.

Not the amplitude itself. Not the cube. The square. Why?

Every attempt to derive the Born rule from first principles has required additional assumptions — decision-theoretic axioms (Deutsch-Wallace), envariance (Zurek), frequency arguments (Hartle-Farhi), or symmetry postulates (Hardy).

Each approach works, but each introduces machinery beyond the bare quantum formalism.

The derivation here is different. It requires no new machinery. The ingredients are: a complex Hilbert space (AP07), definite records (Axiom R), a single break (Axiom B), and distinction (Axiom S).

These force pure-state targets, non-contextuality, and orthogonal additivity. Gleason does the rest. The probability structure is not chosen. It is forced by the axioms and the mathematical structure of the space they act on.

The Born rule is the sound the axioms make when they land on Hilbert space.

§9 — Kill Switches

Global numbering note: Kill switch numbers are globally unique across the corpus.

KS-55 — Contextuality (EMPIRICAL). Proposition 2 derives Gleason non-contextuality from Axiom R + Axiom B.

If a reproducible, loophole-free experiment demonstrates contextual outcome probabilities — that is, if the probability of outcome a when measuring A depends on which compatible observable is measured alongside A, in a way not accounted for by standard QM — then Proposition 2 fails and the Born rule derivation is threatened.

Note: this concerns Gleason non-contextuality of outcome probabilities, not Kochen-Specker contextuality of value assignments. The axioms deny pre-existing values (Axiom R), so KS contextuality is expected and is not a threat.

Status: LIVE — EMPIRICAL. Structurally secure.

KS-56 — Non-Born statistics (EMPIRICAL). Theorem 1 derives the Born rule as the unique probability measure. If a physical system were found whose measurement statistics violate the Born rule, Theorem 1 would fail.

This would require either: (a) a violation of Gleason's theorem (impossible — mathematical theorem), (b) a genuinely fundamental (not merely effective) physical Hilbert space of dimension < 3 with outcome probabilities not inherited from an embedding in the full physical Hilbert space (excluded by AP10: physical wavefunctions live in $L^2(R^3)$, which is infinite-dimensional), or (c) a physical situation where actualisation is contextual (tested by KS-55).

Status: LIVE — EMPIRICAL. All experiments confirm Born rule statistics to high precision. Structurally secure.

KS-57 — Mixed-state actualisation (EMPIRICAL). Proposition 1 derives that record targets are pure states (extremal points).

If a physical process were demonstrated in which a mixed state (a non-extremal state, not a pure state embedded in a larger pure state) is actualised directly — that is, if the universe writes a record that is inherently ambiguous about which pure state obtains — Proposition 1 and Axiom R's definiteness requirement would fail.

Status: LIVE — EMPIRICAL. No known physical process actualises mixed states directly. All observed measurements produce definite outcomes consistent with pure-state collapse. Structurally secure.

§10 — Conclusion

The probability structure of actualisation is derived from the axioms.

Axiom R requires definite records. Definite means extremal — no residual convex ambiguity. In Hilbert space, extremal states are pure states. Mixed states admit multiple decompositions and are not definite.

Therefore record targets are pure states (Proposition 1).

Axiom R writes a record for the state being actualised. Axiom B provides one break. The axioms supply exactly two inputs to the actualisation event: the state and the projector.

The record registers what happened, not what could have happened. Context is not recorded because context is not an event. Therefore actualisation is Gleason non-contextual (Proposition 2).

Coarse-graining of mutually exclusive outcomes respects additivity (Lemma 1).

The probability measure is a frame function on a complex, separable Hilbert space of dimension ≥ 3. Gleason's theorem forces the Born rule as the unique such measure. $\mu(P) = \mathrm{Tr}(|\psi\rangle\langle\psi| \cdot P)$ is the only possibility (Theorem 1).

**The Born rule is not imported. It is not assumed. It is not postulated. It is forced by the axioms acting on the mathematical structure of the pre-state. There is no other measure.

There was never a choice.**

You have spent your life experiencing probabilities — every coin flip, every dice roll, every uncertain moment.

The quantum version of probability — the Born rule — governs every one of those events at the deepest level. And it was never a postulate. It was never a guess.

It was the only measure the structure of reality admits. The axioms land on Hilbert space. The Born rule is what comes out. Not because anyone chose it. Because nothing else can.

Claim Summary

Derived: Record targets are pure states / extremal points (Proposition 1, from Axiom R + AP07). Actualisation is Gleason non-contextual (Proposition 2, from Axiom R + Axiom B).

Orthogonal additivity (Lemma 1, from Axioms S + B + R).

Born rule is the unique probability measure on outcomes of actualisation (Theorem 1, from Propositions 1 + 2 + Lemma 1 + AP07 + AP10 + Gleason's theorem). AP07 Born rule retroactively grounded.

AP23 upstream caps closed: pure-state targets derived, CHSH $S = 2\sqrt{2}$ fully axiomatic.

Mathematical tool: Gleason's theorem (1957). Mathematical theorem about Hilbert spaces. Not a physical postulate. Converts physical hypotheses (Propositions 1 + 2 + Lemma 1) into the Born rule.

Conditional on: Nothing. EH and QRA proved in AP20. Axioms unconditional.

Depends on: Axiom S (distinction, non-trivial outcome space), Axiom B (single break), Axiom R (definite, irreversible record), AP07 (complex Hilbert space), AP10 (three spatial dimensions, $L^2(R^3)$ separable and infinite-dimensional, dim ≥ 3).

What is closed: Born rule gap (AP07). Pure-state record target gap (AP23). Bell prediction fully axiomatic (AP23).

Formal results: Proposition 1 (Record Target, DERIVED). Proposition 2 (Non-Contextuality, DERIVED). Lemma 1 (Orthogonal Additivity, DERIVED). Theorem 1 (The Measure — Born Rule, DERIVED).

Kill switches: KS-55 (Gleason non-contextuality, EMPIRICAL, LIVE). KS-56 (non-Born statistics, EMPIRICAL, LIVE). KS-57 (mixed-state actualisation, EMPIRICAL, LIVE).

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