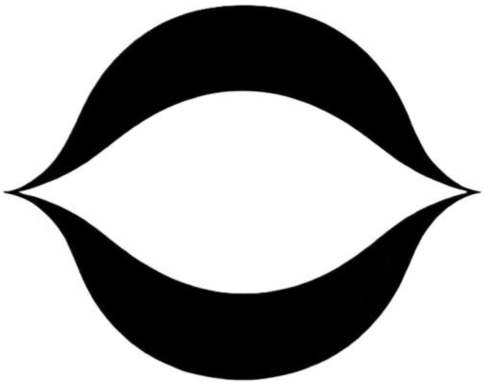

The Harmonics

Artist's Proof 27

Gauge Structure

The electroweak gauge group from the two-sector axiom

Status and Dependency

This paper derives the electroweak gauge group SU(2) × U(1) from Axiom S — the two-sector structure $\mathcal{L}/\mathcal{P}$ connected by the involution σ.

The break ε, as the boundary between sectors, carries an internal two-component state on the manifold encoding its relationship to both sectors.

Local basis freedom on this internal space generates a gauge connection with structure group U(2) ≅ SU(2) × U(1).

Four formal results. Lemma 1 derives the internal electroweak state space $\mathcal{H}\_EW \cong \mathbb{C}^2$ from Axiom S. Lemma 2 derives the gauge freedom from local basis invariance.

Proposition 1 decomposes U(2) → SU(2) × U(1) and identifies weak isospin and hypercharge. Proposition 2 provides the structural argument for chiral coupling selection (why left-handed fields form doublets and right-handed fields are singlets).

Proposition 2 is tagged STRUCTURAL: physically motivated and consistent with all data, but the coupling theorem is not yet formally proved.

The dependency chain: Axiom S (two sectors $\mathcal{L}/\mathcal{P}$) → Axiom B (single break between sectors) → AP07 (complex Hilbert space) → AP25 (Born rule, inner product physical) → AP11 (spin-½ from SU(2) cover of SO(3)) → AP15 (U(1) from phase freedom) → AP19 (SU(3) from orientation freedom) → this paper (internal sector-relationship freedom → SU(2) × U(1)).

Epistemic status per section. §1 (The Missing Group): historical. §2 (The Internal State Space): derived — Lemma 1. §3 (Gauge Freedom): derived — Lemma 2. §4 (The Electroweak Decomposition): derived — Proposition 1. §5 (Chiral Coupling): structural (principal debt) — Proposition 2, consistent with all data, formal coupling theorem not exhibited.

Debts and scope. This paper derives the electroweak gauge group and identifies the chiral coupling pattern.

It does NOT derive: (i) exact hypercharge assignments for each fermion species, (ii) the generation structure (why three generations), (iii) the Higgs mechanism or electroweak symmetry breaking, (iv) Yukawa couplings, fermion masses, or mixing matrices, (v) anomaly cancellation conditions.

Notation

$\mathcal{L}$, $\mathcal{P}$ — the two sectors of Axiom S, connected by the involution $\sigma: \mathcal{L} \leftrightarrow \mathcal{P}$.

ε — the break. The minimal asymmetry. The object that connects the two sectors and writes records on the manifold.

$\mathcal{H}$_EW — the internal electroweak state space of ε. $\mathcal{H}$_EW ≅ $\mathbb{C}^2$. Not spin space (AP11). Not colour space (AP19). The space of ε's relationship to the two-sector structure.

$|\mathcal{L}\rangle$, $|\mathcal{P}\rangle$ — a basis for $\mathcal{H}$_EW. Abstract internal labels, not spacetime directions.

U(2) — the group of 2×2 unitary matrices. The full gauge group on $\mathcal{H}$_EW before decomposition.

SU(2) — the traceless (relative orientation) part of U(2). Candidate for weak isospin.

U(1)_Y — the determinant part of U(2). Candidate for hypercharge.

U(1)_EM — electromagnetic gauge symmetry (AP15). The post-breaking remnant: $Q = T^3 + Y/2$.

Chirality — a Lorentz property of spinor fields. Left-chiral (ψ_L) and right-chiral (ψ_R) are eigenstates of γ^5.

Kill Switches

KS-61 (State space existence): LIVE — STRUCTURAL.

KS-62 (Gauge group overcounting): LIVE — STRUCTURAL.

KS-63 (Chiral coupling derivation): LIVE — STRUCTURAL. Principal debt.

KS-64 (SU(2)_weak ≠ SU(2)_spin): LIVE — STRUCTURAL.

KS-65 (Hypercharge assignments): OPEN DEBT.

KS-66 (Generations): OPEN DEBT.

KS-67 (AP15/AP27 EWSB consistency): OPEN DEBT.

Here is how to destroy this paper.

Show that no well-defined internal $\mathbb{C}^2$ degree of freedom arises from the two-sector structure — that kills Lemma 1. Or show that the local basis freedom generates a gauge group other than U(2) — that kills Lemma 2. Or show that parity is conserved in the weak interaction, contradicting Wu (1957) and all subsequent data — that kills the chiral coupling pattern.

Or show that the derived SU(2) on $\mathcal{H}$_EW is identical to the Lorentz spin SU(2) of AP11, not a separate group — that kills the physical content. The argument hands you these weapons.

§1 — The Missing Group

The Standard Model of particle physics has gauge group SU(3) × SU(2) × U(1). The axioms have derived two of the three factors.

AP15 derives U(1) from phase freedom. The complex amplitude of ε has an overall phase; local rephasing is unphysical; demanding consistency under local rephasing forces a gauge connection with structure group U(1).

This is electromagnetic gauge symmetry.

AP19 derives SU(3) from spatial orientation freedom.

The break ε propagates through a 3-dimensional manifold (AP05, AP10); local reorientation of the three spatial axes is unphysical; demanding consistency under local reorientation forces a gauge connection with structure group SU(3).

This is colour gauge symmetry.

One factor remains: SU(2). In the Standard Model, this is weak isospin — the internal symmetry that governs the weak nuclear force. It acts only on left-handed fermion fields, organising them into doublets.

Right-handed fields are singlets: the weak force does not see them.

The pattern of the previous derivations points to the answer. Each gauge group arises from a physical freedom in how ε is described:

Phase freedom → U(1). Orientation freedom → SU(3). ??? freedom → SU(2).

What degree of freedom has not yet been gauged? The axioms provide one structural feature that has not yet been exploited for gauge purposes: Axiom S. The two-sector structure $\mathcal{L}/\mathcal{P}$.

You have heard three notes of a chord. Two are identified — U(1) and SU(3). The third is sounding but unnamed. This paper names it.

§2 — The Internal State Space

Axiom S states that reality has two sectors, $\mathcal{L}$ and $\mathcal{P}$, connected by the involution $\sigma: \mathcal{L} \leftrightarrow \mathcal{P}$.

The break ε is the boundary between them — the object that connects the two sectors and, through Axiom R, writes records on the manifold.

The break is not purely in $\mathcal{L}$. It is not purely in $\mathcal{P}$. It is the boundary: the thing that sits between and connects them.

When ε propagates on the manifold as a quantum field, it necessarily carries information about its relationship to both sectors.

This relationship is an internal degree of freedom. It is not spatial position (governed by Axiom C). It is not spin (the Lorentz spinor structure, AP11). It is not colour (spatial orientation freedom, AP19).

It is not overall phase (the U(1) of AP15). It is the one remaining structural feature: the break's relationship to the two sectors.

Lemma 1 (Internal Electroweak State Space). The break ε, as the boundary between sectors $\mathcal{L}$ and $\mathcal{P}$ (Axiom S), carries an internal state on the manifold encoding its relationship to each sector.

The space of internal states is $\mathcal{H}\_EW \cong \mathbb{C}^2$.

Proof. (i) Axiom S provides exactly two sectors: $\mathcal{L}$ and $\mathcal{P}$. No more, no fewer. The break connects them.

- On the manifold, a propagating field arising from ε carries a state that encodes how ε relates to each sector.

Since there are two sectors, the state has two components: one for ε's relationship to $\mathcal{L}$, and one for ε's relationship to $\mathcal{P}$.

- Why is this a complex linear vector space?

The components must support superposition: the pre-state is a complex Hilbert space (AP07), and every quantum degree of freedom is represented as a vector in a Hilbert space.

The sector-relationship components are quantum degrees of freedom — they participate in interference, entanglement, and the Born rule (AP25). A discrete label (e.g., "ℒ or 𝒫" with no superposition) would not support interference.

A real vector space would not support the complex phase structure required by AP07. Therefore the internal state space is a two-dimensional complex Hilbert space.

- The state is a vector (z_ℒ, z_𝒫) ∈ $\mathbb{C}^2$. The inner product on ℋ_EW is physical: it determines transition amplitudes via the Born rule (AP25). The internal state space is ℋ_EW ≅ $\mathbb{C}^2$.

- This space is internal: not spacetime, not spin, not colour, not phase. It is the space of the break's internal orientation relative to the two-sector structure. ■

Note: Why exactly two components and not more? Because Axiom S provides exactly two sectors. Three sectors would give $\mathbb{C}^3$. n sectors would give $\mathbb{C}^n$. Two sectors give $\mathbb{C}^2$.

The same logic as AP19: three spatial dimensions give $\mathbb{C}^3$ for colour.

Note: States in ℋ_EW are not merely classical labels "ℒ or 𝒫." The break can exist in coherent superpositions of |ℒ⟩ and |𝒫⟩. A general state α|ℒ⟩ + β|𝒫⟩ is physically meaningful.

Such superpositions exhibit interference. This is what gives rise to the full continuous U(2) gauge freedom rather than a discrete Z_2 swap.

Note: ℋ_EW is distinct from all previously derived spaces.

In fibre bundle language: the spinor bundle, the colour bundle, and the electroweak bundle are three distinct fibre bundles over the manifold, with structure groups SU(2)_spin, SU(3), and U(2) respectively.

Their fibres are different vector spaces. Their connections are independent.

§3 — Gauge Freedom

At each point x on the manifold, the propagating field carries an internal state in $\mathcal{H}$_EW ≅ $\mathbb{C}^2$. Is the choice of basis physical?

The two basis vectors $|\mathcal{L}\rangle$ and $|\mathcal{P}\rangle$ label the break's relationship to each sector. The involution σ maps $\mathcal{L} \leftrightarrow \mathcal{P}$, so the labelling is ambiguous: σ swaps the basis.

This is a discrete (Z_2) ambiguity. But the full gauge redundancy is larger than Z_2.

All physical observables on $\mathcal{H}$_EW depend on inner products: transition amplitudes are $|\langle\varphi|\psi\rangle|^2$ (Born rule, AP25), and expectation values are $\langle\psi|A|\psi\rangle$. Any transformation on $\mathcal{H}$_EW that preserves all inner products is physically undetectable.

The group of inner-product-preserving transformations on $\mathbb{C}^2$ is U(2). The involution σ is one element of U(2). But U(2) contains all inner-product-preserving transformations, not just the swap.

Axiom S gives the two sectors equal standing — the 1:1 — and no axiom introduces a preferred basis or subgroup of U(2) on $\mathcal{H}$_EW.

Lemma 2 (Electroweak Gauge Freedom). The choice of basis on $\mathcal{H}$_EW at each point of the manifold is unphysical. Demanding consistency under local basis changes forces a gauge connection with structure group U(2).

Proof. (i) A local basis change on $\mathcal{H}$_EW at point x preserves the inner product on $\mathbb{C}^2$ (the inner product is physical: Born rule, AP25). The group of such transformations is U(2).

The discrete involution $\sigma \in$ U(2) motivates the basis ambiguity; the Born rule promotes it from discrete to continuous by requiring invariance under all inner-product-preserving transformations.

- No additional structure from the axioms reduces U(2) to a proper subgroup. The physical observables on $\mathcal{H}$_EW are exhausted by inner-product structure. No further invariant structure is introduced by the axioms.

- If the basis choice varies from point to point — if $U(x) \in U(2)$ depends on position — then partial derivatives of the field pick up extra terms.

A compensating gauge connection A_μ(x) must be introduced. This is the standard construction of a non-Abelian gauge theory.

- The logic is identical to AP15 (phase → U(1)) and AP19 (orientation → SU(3)). Here: sector-relationship freedom → local U(2) invariance → electroweak gauge connection. ■

Note: U(2) has four generators (the 2×2 Hermitian matrices form a 4-dimensional real vector space). Four gauge bosons. The count matches the Standard Model's electroweak sector (W^1, W^2, W^3, B before symmetry breaking).

You have watched the same mechanism three times now. Identify a freedom. Demand local invariance. Force a gauge connection. Phase → U(1). Orientation → SU(3). Sector-relationship → U(2).

The Standard Model's gauge structure is not arbitrary. It is the axioms' internal geometry, projected onto the manifold one freedom at a time.

§4 — The Electroweak Decomposition

The gauge group U(2) is not simple. It decomposes into two factors.

Proposition 1 (Electroweak Decomposition). U(2) on ℋ_EW decomposes as U(2) ≅ SU(2) × U(1)_Y. SU(2) acts on the relative orientation of the two sector-components (weak isospin).

U(1)_Y acts on the overall phase of the doublet (hypercharge).

Proof. Every unitary matrix U ∈ U(2) can be uniquely written as U = e^{iα} · V, where α ∈ ℝ and V ∈ SU(2) (det(V) = 1).

The factor e^{iα} is the determinant of U: det(U) = e^{2iα}. For the Lie algebra: u(2) ≅ su(2) ⊕ u(1).

The SU(2) factor acts on the relative orientation of the two components in ℋ_EW. It mixes |ℒ⟩ and |𝒫⟩ while preserving their total norm and the determinant.

Three generators: the Pauli matrices σ_1, σ_2, σ_3 (equivalently, the weak isospin operators T^1, T^2, T^3).

The U(1) factor is the overall phase: multiplication by e^{iα} on both components simultaneously. One generator: hypercharge Y.

This U(1)_Y is NOT the electromagnetic U(1)_EM of AP15. The electromagnetic charge is a combination: $Q = T^3 + Y/2$. AP15 derived the post-breaking remnant.

This paper derives the pre-breaking structure from which it emerges after electroweak symmetry breaking. ■

Note on global structure: The Lie group isomorphism is U(2) ≅ (SU(2) × U(1)) / $\mathbb{Z}_2$. This global structure constrains allowed representations and is part of hypercharge quantisation.

This paper derives only the local gauge algebra. Global structure, allowed representations, and anomaly constraints are open debts.

Note on hypercharge assignments: The decomposition identifies U(1)_Y as hypercharge, but the specific hypercharge values for each fermion species are not derived here.

These must come from the detailed representation content of ε's internal structure and anomaly cancellation. See KS-65.

Note on AP15 consistency: AP15's U(1) is the electromagnetic remnant after symmetry breaking. AP27's U(1)_Y is hypercharge before breaking. Their consistency requires a symmetry-breaking mechanism the axioms have not yet derived.

This is part of debt (iii) in the scope statement.

§5 — Chiral Coupling Selection

Epistemic status: STRUCTURAL (PRINCIPAL DEBT). The following argument is physically motivated, consistent with all known data, and grounded in the axioms. The coupling theorem is not formally proved. This is the paper's principal open problem.

The electroweak gauge group SU(2) × U(1)_Y has been derived. But the Standard Model has a remarkable feature: SU(2) does not couple equally to all fermion fields.

It couples to left-chiral fields as doublets and to right-chiral fields as singlets. The weak force is maximally parity-violating. Why?

Chirality vs helicity. Chirality is a Lorentz-representation property. A Dirac spinor Ψ decomposes into left-chiral and right-chiral Weyl components via P_L = ½(1 − γ^5) and P_R = ½(1 + γ^5).

These transform under independent representations of the Lorentz group: Ψ_L ∈ (½, 0) and Ψ_R ∈ (0, ½).

The axioms have one structural asymmetry not yet deployed: Axiom R's arrow of time.

Proposition 2 (Chiral Coupling Selection). The gauge connection on ℋ_EW couples to left-chiral fermion fields as doublets and to right-chiral fermion fields as singlets.

Argument. The break ε has a direction: it proceeds from the 1:1 symmetry (pre-state) to the manifold (record). This defines a fundamental arrow (Axiom R).

On the Lorentzian manifold (AP05), this arrow selects a preferred time-orientation, which selects a preferred decomposition of the Lorentz group into its two chiral half-representations.

The involution σ maps ℒ ↔ 𝒫. By AP22, σ acts as CPT on the manifold. CPT conjugation interchanges P_L and P_R: it maps left-chiral fields to right-chiral fields.

Axiom R breaks CPT as a dynamical symmetry — the break goes forward in time, not backward. The two chiral projections are NOT dynamically equivalent.

For a left-chiral field Ψ_L, the arrow of time and the chiral structure are anti-aligned. The two internal components $z_\mathcal{L}$ and $z_\mathcal{P}$ are dynamically independent.

The field carries the full $\mathbb{C}^2$ and transforms as a doublet under SU(2).

For a right-chiral field Ψ_R, the arrow of time and the chiral structure are aligned. The alignment locks the two sector-components together. The internal $\mathbb{C}^2$ collapses to a single effective degree of freedom.

The field is a singlet under SU(2).

What this argument needs to become a theorem. (a) Define D_μ on the tensor product bundle (spinor $\otimes$ $\mathcal{H}_{EW}$).

(b) Show the coupling to $P_L\Psi$ is non-trivial (full $\mathbb{C}^2$) while the coupling to $P_R\Psi$ vanishes (reduces to Abelian U(1)).

(c) Derive this from Axiom R, the Lorentz structure (AP05, AP11), and the CPT identification (AP22) without importing the Standard Model's chiral coupling.

Steps (a) and (b) are well-defined mathematical problems. Step (c) is the hard part.

If the coupling theorem cannot be proved, the paper still derives $SU(2) \times U(1)_Y$ as the electroweak gauge group (Lemma 1 + Lemma 2 + Proposition 1), but the chiral coupling reverts to structural identification rather than derivation.

You have watched the axiom's arrow — the irreversibility of records — reach into the internal structure of the gauge field and break the symmetry between left and right.

The weak force does not see right-handed particles because the arrow of time has locked their internal doublet into a singlet. Parity violation is not a quirk of nature.

It is Axiom R, projected onto the electroweak fibre.

§6 — The Deeper Point

Synthesis note: non-load-bearing.

The Standard Model has three gauge groups. The axioms derive all three. Each arises from a different structural freedom of the break:

Phase freedom (how ε's complex amplitude is described) → U(1).

Orientation freedom (how ε's spatial axes are described) → SU(3).

Sector-relationship freedom (how ε's relationship to $\mathcal{L}/\mathcal{P}$ is described) → SU(2) × U(1)_Y.

Three freedoms. Three gauge groups. One axiom set. The Standard Model's gauge structure is the shape of the axioms projected onto the manifold. The harmonics were always sounding.

§7 — Kill Switches

Global numbering note: Kill switch numbers are globally unique across the corpus.

KS-61 — Internal state space existence (STRUCTURAL). If no well-defined internal ε-state space can be constructed from Axiom S, Lemma 1 fails and AP27 collapses.

The construction follows the same pattern as AP15 and AP19. Status: LIVE — STRUCTURAL.

KS-62 — Gauge group overcounting (STRUCTURAL). If the local basis freedom generates a group other than U(2) — larger or smaller — Lemma 2 and Proposition 1 fail. Status: LIVE — STRUCTURAL.

KS-63 — Chiral coupling derivation (STRUCTURAL/EMPIRICAL). If the chiral coupling cannot be formally derived, the paper derives the gauge group but not the representation content.

If experiment showed parity conservation in the weak interaction (contradicting Wu 1957 and all subsequent data), the chiral pattern itself would fail. Status: LIVE — STRUCTURAL. Principal open problem.

KS-64 — SU(2)_weak ≠ SU(2)_spin conflation (STRUCTURAL). If the derived SU(2) on ℋ_EW is shown to be identical to the Lorentz spin SU(2) of AP11, the paper has derived the wrong group.

Lemma 1 explicitly constructs ℋ_EW as an internal space distinct from spin space. Status: LIVE — STRUCTURAL.

KS-65 — Hypercharge assignments (OPEN DEBT). Specific hypercharge values for each fermion species not derived.

KS-66 — Generations (OPEN DEBT). Why three generations of fermions exist is not derived.

KS-67 — AP15/AP27 EWSB consistency (OPEN DEBT). The relationship between AP15's U(1) and AP27's U(1)_Y requires electroweak symmetry breaking.

§8 — Conclusion

The electroweak gauge group is derived from Axiom S.

The break ε carries an internal two-component state encoding its relationship to the two sectors: $\mathcal{H}$_EW ≅ $\mathbb{C}^2$ (Lemma 1). Local basis invariance forces a gauge connection with structure group U(2) (Lemma 2).

The algebraic decomposition U(2) ≅ SU(2) × U(1)_Y identifies weak isospin and hypercharge (Proposition 1).

The chiral coupling pattern is structurally identified through Axiom R's arrow of time and the CPT identification (Proposition 2, structural, principal debt).

The Standard Model's gauge structure SU(3) × SU(2) × U(1) is now derived: AP15 (U(1) from phase freedom), AP19 (SU(3) from orientation freedom), AP27 (SU(2) × U(1)_Y from sector-relationship freedom).

Three gauge groups from three structural freedoms of ε.

Claim Summary

Derived: Internal electroweak state space $\mathcal{H}$_EW ≅ $\mathbb{C}^2$ from Axiom S (Lemma 1). Gauge freedom from local basis invariance, structure group U(2) (Lemma 2). Electroweak decomposition U(2) ≅ SU(2) × U(1)_Y (Proposition 1).

Structural: Chiral coupling selection — left-handed doublets, right-handed singlets (Proposition 2). Consistent with data. Formal coupling theorem not exhibited.

Conditional on: Nothing. EH and QRA proved in AP20. Axioms unconditional.

Depends on: Axiom S, Axiom B, Axiom R, Axiom C, AP05, AP07, AP11, AP15, AP17, AP19, AP22, AP25.

Formal results: Lemma 1 (DERIVED). Lemma 2 (DERIVED). Proposition 1 (DERIVED). Proposition 2 (STRUCTURAL).

Kill switches: KS-61 through KS-67.

Open debts: Hypercharge assignments, generation structure, Higgs mechanism, Yukawa couplings, CKM/PMNS mixing, anomaly cancellation.

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