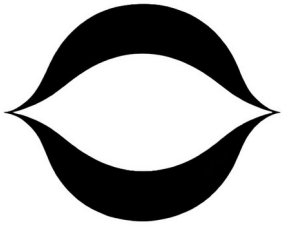

The Connection

Artist’s Proof 15

Forces and Constants

Electromagnetism as non-disconnection on a local manifold

Contents

Artist’s Note

What this paper does — and why.

This is Artist’s Proof 15 of The 420 Code. It is the second chapter of Notebook IV (Forces and Constants). AP06 opened the Notebook by grounding the 420 Code’s ε in the leakage ratio η that finite c enforces at every boundary. AP15 takes the next step: it derives electromagnetism.

The argument is a single move. The Born rule of quantum mechanics (derived in AP09 from the two-sector structure of Axiom S) is invariant under a phase rotation: $|e^{i\theta}\psi|^2 = |\psi|^2$. The phase is U(1). The phase is global at the pre-state level — the 1:1 has no points, no “here” versus “there”, no structure across which the phase could vary. The phase is one number, everywhere. Then records are written. The manifold M emerges with locality (AP20). The phase, still one at the pre-state level, must now be read at each point of a manifold that has points. The connection Aμ is the field that does that reading. Its curvature Fμν is the electromagnetic field. The phase was always global. The manifold made it look local. The connection is what global looks like through local eyes.

Five theorems and one corollary carry the derivation. Theorem 1 (ε is charged): the electron carries non-zero U(1) charge because Axiom B puts it outside the involution’s domain — the electron has no mirror image, that is the entire reason. Theorem 2 (ε forces non-trivial connection): gauge invariance plus $q \neq 0$ forces minimal coupling $\partial\mu \to D\mu$; the field equations then force $F\mu\nu \neq 0$ wherever ε is present. Theorem 3 (action principle from path sum): the action principle is already inside the path sum derived in AP09. Theorem 4 (Phase Coherence Partition): the pre-state’s phase coherence partitions exhaustively into connection curvature on M and entanglement in $\not{H}$, with ε as the sole transfer mechanism — the connection and entanglement are not two structures but two columns in one ledger. Theorem 5 (Charge Quantisation): U(1) is compact, Peter–Weyl gives integer representations, ε is the minimum element — charges are integer multiples of e.

Four kill switches engage. KS-28 (phase-group uniqueness) and KS-29 (Maxwell uniqueness) are closed mathematically. KS-30 (phase globality) is LIVE — STRUCTURAL: divide the pre-state and the connection is not forced to exist. KS-31 (entanglement–connection identification) is held LIVE — STRUCTURAL: Theorem 4 establishes the partition by exhaustion and identifies ε as sole

transfer mechanism, but does not exhibit a conserved balance equation between connection curvature and entanglement; the identification is a structural reading of strong correspondence, not a closed theorem with a commensurable conserved quantity. An existing corpus kill switch, KS-4 (α ≈ 1/137), remains LIVE — HARD: AP15 derives that ε couples to the connection with some strength; the numerical value of the coupling is the work of AP24 and AP28.

All open gaps in AP15's scope close in this paper. Gap A (the value of α_em) remains open, carried by KS-4 and addressed in subsequent NB IV chapters. Within AP15's scope, the derivation of classical electromagnetism from the axioms is complete.

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Orientation

Where AP15 fits

The 420 Code is organised across eight bound Notebooks. AP15 is the second chapter of Notebook IV (Forces and Constants), following AP06 (The Leakage Constant) which opens the Notebook by grounding ε in the leakage ratio that finite c enforces.

AP15 depends on Notebook I (Premise) for the axiom 1:1 + 1×ε @ AS and the four conditions {S, B, R, C}; on Notebook II (Spacetime) for the constants c and G, the manifold structure (AP08), and the spatial-dimension count (AP10 — D = 4); on Notebook III (Quantum Mechanics) for the complex Hilbert space $\mathcal{H}$, the Born rule, the Schrödinger evolution, the path sum (AP09), the fermion/boson distinction (AP11), the uncertainty principle and $\hbar$ (AP12), and the decoherence apparatus (AP13); and on AP06 within Notebook IV for the Landauer bound and the leakage framework that the Theorem 4 partition closes against.

AP15 feeds the rest of Notebook IV. The U(1) connection it derives is the canonical example of a gauge structure; AP27 extends the internal-state-space treatment; AP16 breaks U(1)×SU(2) at the electroweak scale; AP19 derives SU(3) colour from the gauge freedom of the break direction; AP24 formalises the multi-projection structure of ε that determines coupling constants; AP14 treats Planck-scale corrections; AP28 derives G via channel-counting using ε = α_em from the chemical-biological boundary AP06 grounds.

Reading order within AP15

Front to back. Section 1 establishes the phase as a derived consequence of the Born rule — U(1) is not imposed, it is already there. Section 2 names the central fact: the pre-state is one. Section 3 performs the central move: the connection is what global truth looks like through local language. Section 4 derives electromagnetism in the usual sense — field strength, gauge invariance, source, Maxwell’s equations. Section 5 derives the Phase Coherence Partition: connection and entanglement are two readings of one undivided pre-state. Section 6 treats charge and its quantisation. Section 7 derives the photon’s three core properties (boson, massless, propagates at c). Section 8 summarises the derivation chain.

How to read this paper

The paper carries two voices. The structural narrative speaks directly — the central facts (the pre-state is one; the connection is the non-disconnection; entanglement and field are two columns in one ledger) are stated in plain language and held visible across the paper. The formal sections speak in the precision the structure requires. Formal blocks open with a bold label (Theorem, Proposition, Definition, Corollary, Kill Switch) and close with the box mark ■. Where the argument depends on a result derived elsewhere in the corpus, the provenance is named at the point of use.

A note on notation

U(1) is the group of phase rotations e^{iθ} — the symmetry of a circle. $\mathcal{H}$ is the complex Hilbert space derived in AP09. M is the Lorentzian manifold derived in AP20 with dimension D = 4 (AP10) and signature (−, +, +, +). Aμ is the U(1) gauge connection — the 1-form on M that encodes the pre-state's phase coherence as read locally. Fμν = ∂μAν − ∂νAμ is the electromagnetic field tensor (the connection's curvature). The covariant derivative Dμ = ∂μ − iqAμ. The current Jμ is the pushforward to M of the ε-record worldline.

Two structural quantities carry corpus-wide meaning. ε is the unique unpaired element of Axiom B — in AP06 identified with the leakage ratio η, here identified as the source of the connection and (by The Lock, Edition 04) with the electron. σ is the involution of Axiom S — in AP09 section 3.2 identified with complex conjugation on $\mathcal{H}$. The phase rotation acts as ψ → e^{iqθ}ψ; the conjugate transforms as ψ* → e^{−iqθ}ψ* — with charge −q.

0 — Dependency and Scope

0.1 — What this paper does

AP15 derives electromagnetism from the axioms. The argument is: the Born rule (AP09) has a U(1) phase symmetry; the pre-state is one (no manifold structure); records emerge a manifold M with locality (AP20); the pre-state's global phase must be read locally; the U(1) connection Aμ is the field that does that reading; its curvature Fμν is the electromagnetic field; ε — the break, the electron — is the source.

The paper closes five structural questions. Theorem 1: ε carries non-zero charge (because Axiom B places it outside the involution's domain). Theorem 2: ε forces non-trivial connection and curvature (by gauge invariance and the field equations). Theorem 3: the action principle is contained in the path sum (AP09). Theorem 4: the pre-state's phase coherence partitions exhaustively into connection curvature on M and entanglement in $\mathcal{H}$, with ε as sole transfer mechanism. Theorem 5: charge is quantised in integer multiples of ε's charge.

What the paper does not close: the numerical value α_em ≈ 1/137 (Gap A / KS-4). The fine-structure constant's value is the work of AP24 (Residual) and AP28 (Constant) in subsequent NB IV chapters.

0.2 — Dependencies

AP15 depends on the following derived results.

Notebook I (Premise). The four axioms {S, B, R, C} and the governing axiom 1:1 + 1×ε @ AS. Load-bearing throughout.

AP05 (The Break). The structural derivation of ε as the unique unpaired element. Load-bearing for ε's structural role as source of the connection.

AP06 (The Leakage Constant). The Landauer bound (ε's thermodynamic receipt of k_B T ln 2 per record-writing event, AP06 section 4) and the structural ground for ε ≡ η. Load-bearing for Theorem 4 part (iii) and for the receipt account in the Phase Coherence Partition.

AP08 (The Identity). Einstein's field equations and the Lovelock-type uniqueness argument for gravity. Load-bearing for the parallel uniqueness argument in section 4.4 that selects the Maxwell action as the unique gauge-invariant local action at leading derivative order.

AP09 (Quantum Mechanics). The complex Hilbert space $\mathcal{H}$, complex amplitudes, the Born rule ($P = \psi\psi^*$), the Schrödinger equation, the path sum, and the involution σ identified with complex conjugation (AP09 section 3.2). Load-bearing for the U(1) phase symmetry (section 1) and the action principle (Theorem 3).

AP10 (The Dimension). D = 4 (three spatial + one temporal). Load-bearing for the Lovelock-type uniqueness argument (Maxwell action is dimension-4 gauge-invariant local; the dimension is derived, not assumed).

AP11 (Spin). Fermion/boson distinction and spin from the two-sector structure. Load-bearing for the photon's identification as a spin-1 boson and for ε as a spin-½ fermion.

AP12 (The Limit). The uncertainty principle and $\hbar$ as the quantum cost per record. Load-bearing for the path sum's $\exp(iS/\hbar)$ weighting.

AP13 (Decoherence). Record-writing produces decoherence; the accumulated record is the environment. Load-bearing for Theorem 4 part (iii)(b): ε-record-events decohere the subsystems that shared phase coherence.

AP14 (Correction). Time-slicing of the path sum (AP14 section 2.3). Load-bearing for Theorem 3: the action principle's derivation from the discrete record algebra via the time-sliced sum.

AP20 (The Proof). The Embedding Hypothesis (EH) and the Quantum–Records Algebra (QRA): the algebraic pre-state structure embeds into the physical manifold M. Load-bearing for the bundle construction in section 3 (the U(1) principal bundle over M) and for the EH pushforward used throughout.

0.3 — Axiom mapping

The four axioms map to AP15 as follows.

Axiom S (two sectors / involution). The two-sector structure of $\mathcal{H}$ gives the involution σ = complex conjugation. σ acts as $\psi \to \psi^*$. The Born rule $P = \psi\psi^*$ is

the structural fingerprint of σ. The Born rule's invariance under ψ → e^{iθ}ψ is the U(1) phase symmetry that this paper derives electromagnetism from.

Axiom B (unique breaking). ε is outside Dom(σ) — the one element with no σ-image. Theorem 1: ε is therefore charged. Theorem 5: ε carries the minimum non-zero charge. The break is the charge. The break is the source of the connection. The electron is charged because it has no mirror image.

Axiom R (record monotonicity). Records are written irreversibly. Time begins. The accumulated record is the environment (AP13). Theorem 3: the action principle is contained in the path sum. Theorem 4: ε-record-events transfer phase coherence from entanglement to field, irreversibly.

Axiom C (Constraint — locality). Finite c. The manifold acquires spatial separation (finite propagation means "here" and "there" are distinguishable). The photon propagates at c — c is not a property of light but the characteristic speed of the wave equation, which is the causal bound of Axiom C read through the derived Maxwell dynamics.

0.4 — Epistemic status per section

1 (The Phase): [DERIVED] — U(1) follows from the Born rule (Axiom S, AP09).

2 (The Pre-state is One): [DERIVED] — the pre-state is structurally unbroken before records; the manifold emerges via R + C + G (AP20). Proposition (Global U(1) covariance) is proved from the Born rule and Schrödinger evolution.

3 (The Connection): [DERIVED USING STANDARD MATHEMATICS] — the bundle construction follows from $\not{H}$'s complex structure (AP09) and M's locality (AP20); standard bundle mathematics applied to the derived manifold.

4 (Electromagnetism): [DERIVED] — Theorems 1, 2, 3 and the Corollary are derived from the axioms. The field equations follow uniquely at leading derivative order via the Lovelock-type argument (parallel to AP08's derivation of gravity).

5 (The Non-actualised): [DERIVED] — Theorem 4 (Phase Coherence Partition) is derived. All premises are derived or established. The partition is exhaustive by construction.

6 (Charge): [DERIVED USING STANDARD MATHEMATICS] — Theorem 5 (Charge Quantisation) uses Peter–Weyl on the compact group U(1) and the bundle construction of section 3.

7 (The Photon): [DERIVED] — boson via AP11; massless via U(1) exactness and gauge-invariance of the mass term; spin 1 via the 1-form structure of Aμ; propagates at c via Axiom C and the Maxwell wave equation.

8 (Summary of Derivation): [SYNTHESIS] — enumeration of the derivation chain.

0.5 — Kill switch summary

Four kill switches engage in AP15. One existing corpus kill switch (KS-4) is affected. Identifiers per the Master Kill Switch Registry.

KS-28 (Phase-group uniqueness (U(1))). CLOSED — MATHEMATICAL. targets the derivation of U(1) as the gauge group.

KS-29 (Maxwell uniqueness). CLOSED — MATHEMATICAL. targets the Maxwell action as the unique dimension-4 gauge-invariant local action.

KS-30 (Phase globality). LIVE — STRUCTURAL. targets the assumption that the pre-state's phase is globally coherent.

KS-31 (Entanglement–connection identification). LIVE — STRUCTURAL. targets the identification of connection curvature and entanglement as two readings of pre-state unity.

KS-4 (α ≈ 1/137). Existing corpus kill switch. AP15 identifies the fine-structure constant as the coupling strength of ε to the connection. Its value remains open. LIVE — HARD. Carried forward to AP24 and AP28.

0.6 — Structural debts owed

AP15 closes the structural debts within its scope. Gap A remains the one open item that AP15 does not close.

Gap A (the value of α_em). AP15 derives that ε couples to the connection with some strength. It does not derive the numerical value ≈ 1/137. This is Open Problem 1 / KS-4. The path to closure runs through AP24 (the six-faces

formalisation of ε's multi-projection structure) and AP28 (the channel-counting derivation of G that uses ε = α_em). [AP15 scope: not closed]

0.7 — Items held open without claiming debt status

The phase coherence partition of Theorem 4 names ε as the sole transfer agent between entanglement and field. Whether other elements of the record algebra can act as transfer agents under specific structural conditions (for example, in the strong-force sector developed in AP19) is held open at the structural level. The expectation from Axiom B (ε is the unique unpaired element) is that no other transfer agent exists; the formal proof of uniqueness across all gauge sectors is not advanced in AP15.

The relationship between the U(1)_EM derived here and the larger gauge structures of subsequent NB IV chapters (SU(2)_L in AP16, SU(3) in AP19) is held open at the bundle-construction level. The expectation is that each gauge group arises by the same structural logic — a global symmetry of the pre-state read through the local manifold — with the gauge group identified by the corresponding symmetry of the algebraic structure. The formal extension lives in AP16, AP19, AP27.

0.8 — Structural relationships

AP05 (The Break). ε is the unique unpaired element. AP15 reads ε as the source of the connection. The break is the charge.

AP06 (The Leakage Constant). AP06 grounds ε in the leakage ratio η. AP15 carries that grounding into the electromagnetic sector: α_em is the leakage ratio at the electromagnetic boundary, the coupling strength of ε to the connection.

AP08 (The Identity). AP08 derives Einstein's field equations via a Lovelock-type uniqueness argument. AP15 uses the same uniqueness argument structure to select the Maxwell action.

AP09 (Quantum Mechanics). AP09 derives $\mathcal{H}$, complex amplitudes, the Born rule, the Schrödinger equation, and the path sum. AP15 reads the Born rule's phase symmetry as U(1) and identifies the path sum as the containing structure for the action principle (Theorem 3).

AP11 (Spin). The fermion/boson distinction follows from the two-sector structure. AP15 reads ε as a fermion (spin ½, Axiom B places it outside Dom(σ)) and the photon as a boson (spin 1, A_μ transforms as a vector under the Lorentz group).

AP13 (Decoherence). Record-writing produces decoherence. AP15's Theorem 4 uses this: ε-record-events decohere the subsystems that shared phase coherence, transferring the coherence from the entanglement column to the field column.

The Lock (Edition 04). The Lock identifies ε with the electron at the chemical-biological scale. AP15 reads charge $-e$ (the electron) as ε itself, and charge $+e$ (the positron) as the conjugate state in the second sector.

AP20 (The Proof). The Embedding Hypothesis (EH) is proved in AP20. AP15 uses the EH pushforward throughout: the algebraic pre-state's phase structure is represented on M as the connection geometry.

AP24 (The Residual) and AP28 (The Constant). KS-4 (the value of α_{em}) is carried forward. AP24 formalises the multi-projection structure of ε; AP28 derives G via channel-counting using $\varepsilon = \alpha_{em} \approx 1/137$. Together they address Gap A.

1 — The Phase

Every quantum state you have ever encountered carries a hidden degree of freedom. It is not the magnitude — the magnitude gives the probability. It is the phase. The angle. The rotation that no measurement can detect directly, but whose consequences shape every interference pattern ever recorded.

AP09 derives complex amplitudes from the Lorentzian signature of the manifold (AP09 section 3.3). The pre-state lives in a complex Hilbert space $\mathcal{H}$. State vectors in $\mathcal{H}$ have complex amplitudes. Every complex amplitude has a magnitude and a phase.

The Born rule (AP09 section 6) gives: P = |ψ|² = ψψ* = Light × Dark. The magnitude determines the probability. The phase does not. Replace ψ with e^{iθ}ψ (where $i^2 = -1$) — rotate the phase by any angle θ — and the probability is unchanged:

$$|e^{i\theta}\psi|^2 = e^{i\theta}\psi \times e^{-i\theta}\psi^* = \psi\psi^* = |\psi|^2$$

This is U(1). The group of rotations of a circle. One parameter: θ. One operation: multiplication by e^{iθ}. The simplest continuous symmetry. And you already have it.

This symmetry is not imposed. It is not a choice. It is a consequence of the Born rule, which is a consequence of the involution σ (Axiom S), which is a consequence of the two-sector structure. U(1) is already derived. It lives in the pre-state. It has been there since AP09.

The question is: what does this symmetry look like when read on the manifold?

2 — The Pre-state is One

Before you can understand the connection, you need to understand what it connects. The answer is: nothing was ever apart.

2.1 — Before records

Before any record is written, there is no manifold. No locality. No “here” vs “there.” The pre-state is the 1:1 — perfect symmetry, undivided, one thing. Superposition is the empty set state (AP09 section 3): 0 and 1 undistinguished.

In this condition, the phase freedom is global in the most absolute sense. There is no structure across which it could vary. There are no points. There is no separation. The phase is one number, everywhere — except there is no everywhere. There is just the one. And you are about to see what happens when a local manifold tries to describe a global truth.

2.2 — Records create locality

Records are written. Axiom R: irreversible, monoid, no inverses. Time begins. The accumulated record is the environment (AP13 section 2.1). The monoid admits embedding into a smooth manifold M (AP20). The manifold has the structure derived in AP10: four dimensions — one temporal (R), three spatial (C, S, B). Lorentzian signature (−, +, +, +).

Locality emerges from R + C + G acting together. R creates time: the record changes, the now does not move (AP09 section 4.3). C creates spatial separation: finite propagation means “here” and “there” are distinguishable. G — the gravitational constant derived in AP08 — is the accumulated record’s effect on the geometry: curvature. Together, they produce a manifold with points, distances, causal structure, and separation. You can point to a location. You can measure a distance. The manifold gives you that.

Now there IS a “here” and a “there.” Now the phase at this point and the phase at that point are, in principle, different questions.

2.3 — But the pre-state was never divided

The manifold has locality. The pre-state does not. The manifold is the accumulated record — the black curve of the eye. It has points, separation, curvature. The pre-state is the unbroken — the white space of the eye. It has no points. It has no separation. Separateness is experienced but not fundamental.

The pre-state remains one. It was one before any records were written. It is one after N records are written. It will be one after N^2 records are written. The records create the experience of separation. The pre-state, which carries the phase, is never separated.

The phase freedom remains global. It was always global. It is still global. The manifold has locality. The phase does not. This is not a contradiction. It is the central fact. Hold it in your mind — everything that follows depends on it.

Proposition (Global U(1) covariance). The path sum K derived in AP09 is covariant under global U(1): K(e^{iθ}ψ_f | e^{iθ}ψ_i) = K(ψ_f | ψ_i) for all θ.

Proof. *The Born rule $P = \psi\psi^*$ is invariant under $\psi \to e^{i\theta}\psi$ (section 1). The Schrödinger evolution operator $\hat{U}$ preserves the inner product structure of $\mathcal{H}$ (AP09 section 4). The path sum K is constructed from products of amplitudes and their conjugates (AP09 section 5, AP14 section 2). Each such product is invariant under a common phase rotation. Therefore K is globally U(1)-covariant.*

The pre-state's phase freedom is not local — it acts identically at every point. This is the mathematical content of "the pre-state is one."

3 — The Connection

Here is the central move of this paper. Watch it carefully — because this is where electromagnetism stops being a separate force and starts being a consequence of unity read through locality.

3.1 — A global symmetry on a local structure

The manifold has points. Each point has a tangent space. The complex structure of $\mathcal{H}$ (AP09) associates to each point a U(1) phase degree of freedom: the fiber of a complex line bundle over M.

At each point, there is a phase. At each point, the phase can be rotated by e^{iθ} without changing the local probability.

If the pre-state were actually local — if the phases at different points were independent — then θ could be different at every point. The symmetry would be local: θ(x), a function on the manifold. This is the standard gauge argument. It assumes phases are independent and then requires a connection to stitch them together.

In the record algebra, the situation is the opposite. The phases are not independent. The pre-state is one. The phase is global. The manifold creates the appearance of separation between phases that were never separated. The connection does not stitch together independent phases. The connection expresses the fact that they were never apart.

3.2 — What the connection is

Definition. The connection Aμ is the field on the manifold that encodes the pre-state's global phase coherence as read at each point of the local structure.

At each point x on the manifold, the pre-state's phase can be read in local coordinates as some angle θ(x). This θ(x) is not the global phase itself — it is a local coordinate representation: a choice of gauge, a section of the bundle.

Different coordinate patches may assign different θ(x) to the same global truth. The connection Aμ is the 1-form that relates these local readings across the manifold:

$$D\mu\psi = (\partial\mu - iqA\mu)\psi$$

The covariant derivative Dμ is the rule for transporting the phase from one point to a neighbouring point on the manifold. Aμ is the connection 1-form — not, in general, a gradient of a scalar.

In any single coordinate patch, a gauge choice gives a local representation θ(x), and the connection can be written Aμ = ∂μθ + Āμ where Āμ is the physically meaningful part. But this decomposition is local. Globally, Aμ is a connection on the U(1) principal bundle over M: the bundle whose fiber at each point is the phase circle S^1 inherited from $\mathcal{H}$ (AP09).

In the absence of ε (no break, no source) and with trivial boundary conditions (no incoming excitations), the pre-state's phase is read consistently everywhere. The bundle is trivial — a global section exists — and the connection can be gauged to Aμ = 0 everywhere, provided the ε-free domain is simply connected (the source-free sector of the embedded manifold is contractible by construction; flat holonomy without curvature on non-simply-connected domains is a mathematical possibility but does not arise in the architecture's construction).

Fμν = 0. No electromagnetic field. The phase is one and the manifold reads it without distortion.

(Source-free solutions with Fμν ≠ 0 — i.e. radiation — arise when boundary or initial data carry non-trivial field configurations; these are consistent with Maxwell's equations (section 4.4) and describe the photon (section 7).)

In the presence of ε, the 1:1 is broken at a point. The connection acquires non-trivial curvature: ε, as a charged source (Theorem 1), forces a non-flat connection via the derived field equations (Corollary, section 4.4). Around any loop enclosing ε, the connection carries non-trivial holonomy — the phase, transported around the loop, returns to a different reading. The connection cannot be gauged to zero. Aμ is not a gradient.

Fμν ≠ 0. The electromagnetic field is the curvature forced by the source. [Epistemic status: DERIVED. That ε forces this is proved in section 4.3–4.4: Axiom B (no σ-image) → q ≠ 0 (Theorem 1) → minimal coupling forced (Theorem 2a) → Maxwell equations with Jμ ≠ 0 → Fμν ≠ 0 (Corollary, section 4.4).]

The connection is the dictionary. When the manifold is source-free, the dictionary is trivial (Aμ = 0 in a suitable gauge). When ε is present, the

dictionary must work around the break, and the curvature Fμν is the measure of how hard it must work.

Correspondence: in the mathematics of fiber bundles, a connection with non-trivial holonomy around a source is exactly what a charged particle produces. The argument identifies the source: ε, the one element with no σ-image (Axiom B). The break is the charge.

3.3 — The connection is not a promotion

In the standard gauge argument, one begins with a global symmetry and promotes it to a local symmetry by fiat. The promotion requires introducing a gauge field Aμ to preserve the Lagrangian under local transformations. This works but has no explanation for why the promotion occurs.

In the record algebra, there is no promotion. The phase was always global. The manifold was always local. The connection Aμ exists because a global quantity (the phase) is being read on a local structure (the manifold). It is not that a global symmetry was promoted to local. It is that a global truth is being expressed through a local language.

The connection is the dictionary.

4 — Electromagnetism

The connection exists. The source exists. Now the algebra does what algebra does — it derives the field equations. And you will recognise them. They are Maxwell's equations.

4.1 — The field strength

The connection Aμ is a 1-form on M. Its curvature is the 2-form:

$$F\mu\nu = \partial\mu A\nu - \partial\nu A\mu$$

This is the electromagnetic field tensor. It encodes the electric and magnetic fields. Fμν is the curvature of the connection. It measures the extent to which the global phase, when transported around a closed loop on the manifold, returns to a different reading.

If Fμν = 0 everywhere, the connection is flat and the global phase is read consistently everywhere. If Fμν ≠ 0, the presence of ε (the charged source that forces non-trivial curvature via the derived field equations) creates a discrepancy between the global truth and the local reading. The manifold's geometry provides the arena; ε is the source. That discrepancy is the electromagnetic field. You feel it every time you touch a doorknob in winter.

4.2 — Gauge invariance

The global phase θ is not observable. Only probabilities are observable (Born rule, AP09 section 6). The connection Aμ can be shifted by Aμ → Aμ + ∂μλ(x) for any smooth function λ(x) without changing Fμν:

$$F\mu\nu \to \partial\mu(A\nu + \partial\nu\lambda) - \partial\nu(A\mu + \partial\mu\lambda) = F\mu\nu$$

This is gauge invariance. It is not a postulate. It is the mathematical consequence of a principal bundle connection on a manifold with local charts.

Once EH gives the manifold M with local coordinate patches (AP20), and the phase fiber S^1 at each point forms a U(1) principal bundle (section 3.1–3.2), local sections of this bundle parametrise how the global phase is read in local coordinates. Changes of local section are parametrised by smooth U(1)-valued

functions λ(x) on overlaps — this is standard bundle mathematics, not a physical postulate.

The transformation Aμ → Aμ + ∂μλ is the connection's response to a change of local section: it is the dictionary's response to a change of language. Local gauge redundancy is therefore not a "promotion" of global invariance; it is an automatic feature of expressing a global structure (the phase) through local coordinates (the manifold).

Only the curvature of the connection — the electromagnetic field — is observable. The gauge freedom is the freedom in how you choose to read the unobservable global phase in local coordinates. Different gauges are different dictionaries. The physics is in Fμν, not in Aμ. You cannot measure the gauge. You can only measure the curvature.

4.3 — The source: ε

The electromagnetic field has a source. The source is ε — the break. The electron.

ε is the one element with no σ-image (Axiom B). It breaks the 1:1. On the manifold, ε is a fermion (AP11 section 3.2), with spin ½ (AP11 section 4), mass m_e (The Lock). It is the minimum viable splinter.

Theorem 1 (ε is charged). ε carries non-zero U(1) charge.

Proof. *Axiom S gives the involution σ on the record algebra. σ is total on the paired sector: for every element a in the paired sector, σ(a) exists and σ(σ(a)) = a. ε lies outside the paired sector: Axiom B states that σ(ε) does not exist in the algebra. (σ is an involution on its domain; ε is not in that domain.)*

Definition (charged). *In this framework, an element a of the record algebra is uncharged (q = 0) if and only if a is in the domain of σ and is σ-fixed: σ(a) = a. An element is charged (q ≠ 0) if it is not σ-fixed — either because σ(a) ≠ a, or because σ(a) does not exist (i.e. a is outside Dom(σ)).*

This definition is justified by the representation theory that follows: σ-fixed ⇒ q = 0 is proved below, and the contrapositive (not σ-fixed ⇒ q ≠ 0) is the direction used in Theorem 1.

AP09 section 3.2 identifies σ with complex conjugation on ℋ: σ(ψ) = ψ. For ψ = |ψ|e^{iφ}, complex conjugation reverses the phase: σ(ψ) = |ψ|e^{−iφ}. In the U(1) representation where a state with charge q transforms as ψ → e^{iqθ} ψ, its conjugate transforms as ψ* → e^{−iqθ}ψ*, i.e. with charge −q.*

Therefore: if an element within Dom(σ) is σ-fixed (σ(a) = a), then it must carry charge q = −q, i.e. q = 0. Equivalently, by contrapositive: any element that is not σ-fixed carries q ≠ 0. (The converse — that q = 0 implies σ-fixed — is not required for this argument and is not claimed.) Elements outside Dom(σ) have no conjugate partner and therefore cannot be σ-fixed; they are charged by the Definition above.

Now: ε is outside Dom(σ) (Axiom B: σ(ε) does not exist). By the Definition above, ε is not σ-fixed. Therefore ε is charged: q ≠ 0.

The electron is charged because it has no mirror image. That is it. That is the entire reason.

Theorem 2 (ε forces non-trivial connection and curvature). The U(1) connection on M couples non-trivially to ε. In any region containing ε, Fμν ≠ 0.

Proof by two complementary arguments, one algebraic and one dynamical.

(a) Algebraic argument (minimal coupling forced). *U(1) gauge invariance is derived (section 1, section 4.2). Under a gauge transformation, ψ → e^{iqθ} ψ. With q ≠ 0 (Theorem 1), the ordinary derivative ∂μψ is not gauge-covariant: ∂μ(e^{iqθ}ψ) = e^{iqθ}(∂μψ + iq(∂μθ)ψ) ≠ e^{iqθ}∂μψ. Gauge invariance of the theory requires the replacement ∂μ → Dμ = ∂μ − iqAμ, where Aμ transforms as Aμ → Aμ + ∂μθ. This is minimal coupling. It is not optional — it is forced by gauge invariance and q ≠ 0. You have no choice here. The algebra leaves no room. The connection Aμ must couple to ε.*

(b) Dynamical argument (Fμν ≠ 0). *The action principle is derived from the path sum (Theorem 3, section 4.4). The unique gauge-invariant action at leading derivative order gives Maxwell's equations ∂νF^{μν} = J^{μ} (section 4.4). The current Jμ ≠ 0 wherever ε-records are present (Definition below; Theorem 1). Therefore ∂νF^{μν} ≠ 0, which requires Fμν ≠ 0, in any region containing ε. [Note: this argument uses Theorem 3 and section 4.4, which appear below. The logical dependence is: Theorem 1 → Theorem 2(a) → Theorem 3 → Maxwell → Theorem 2(b). Part (b) is a corollary of the complete chain, not a prerequisite for it.]*

[Epistemic status: DERIVED. Theorem 1 uses Axiom S (σ as involution on paired sector), AP09 (σ = conjugation), and Axiom B (ε outside σ's domain). Theorem 2(a) uses derived gauge invariance and Theorem 1. Theorem 2(b) uses Theorems 1, 2(a), 3, and the derived Maxwell equations. No external physical postulate is imported; standard mathematical reasoning (gauge theory, variational calculus) is applied on the derived manifold.]

Correspondence: in the mathematics of fiber bundles, a connection with non-trivial holonomy around a source is exactly what a charged particle produces. The argument derives the source: ε, the one element with no σ-image (Axiom B). The break is the charge.

Definition. The current Jμ is the pushforward to M of the ε-record worldline. ε carries charge q ≠ 0 (Theorem 1); on the manifold it traces a timelike worldline γ (Axiom C). The 4-current $J^\mu(x) = q\delta^3(x - x_\varepsilon(t))\, dx_\varepsilon^\mu/dt$ is the density and flow of ε-charge at x. [Epistemic status: DEFINITION. The distributional form uses standard mathematical tools (delta measure, worldline parametrisation) applied to the derived manifold. The physical content — that ε carries charge and traces a worldline — is derived (Theorem 1, Axiom C).]

The coupling strength — how strongly ε sources the connection — is the electric charge. That ε couples to Aμ is structural (ε breaks the 1:1; the connection must respond). The dimensionless strength of this coupling, the fine-structure constant $\alpha_{em} \approx 1/137$, is not derived (KS-4, OPEN). It is the coupling strength of one minimum element breaking one minimum symmetry.

4.4 — Maxwell's equations

The dynamics of Aμ are determined by a uniqueness argument paralleling Lovelock.

Theorem 3 (Action principle from path sum). The action principle is contained in the path sum already derived from the axioms.

Proof. *AP09 section 5 derives the path sum K from the axioms: K assigns transition amplitudes between records by summing over all intermediate record sequences. AP09 section 4 derives the Schrödinger equation $i\hbar\partial\psi/\partial t = \hat{H}\psi$ from the same axioms. The Schrödinger equation is the Euler–Lagrange equation of the action $S = \int(i\hbar\psi\partial\psi/\partial t - \psi\hat{H}\psi)\, dt$.*

The converse also holds: the path sum (AP14 section 2.3), which is a time-sliced product of transition matrix elements, has the form K = ∫Dψ exp(iS[ψ]/ħ) in the EH representation (this is Feynman's standard mathematical result: the time-slicing decomposition of quantum amplitudes reproduces the path integral weighted by exp(iS/ħ); AP14 section 2.3 explicitly constructs the time-slicing from the record algebra, and the EH pushforward onto M yields the functional integral as a mathematical representation of the discrete sum — it does not reintroduce physical continuum microstructure below the record scale).

Therefore the action principle is not an external import but is already contained in the path sum derived from the axioms via AP09.

For the electromagnetic field specifically: the connection Aμ is determined by the record configuration (section 3.2); the path sum over records therefore implicitly includes a sum over connection configurations. The weight assigned to each configuration must be (i) gauge-invariant (U(1), section 1), (ii) Lorentz-invariant (AP20), (iii) local (Axiom C: finite causal bound means local interactions), and (iv) of mass dimension 4 (AP10, D = 4). The unique functional satisfying (i)–(iv) at leading derivative order is S_EM = −(1/4) ∫ FμνF^{μν} d⁴x. Uniqueness follows from the same Lovelock-type argument used for gravity (AP08): the constraints select a unique term at lowest derivative order. (Higher-order gauge-invariant terms are consistent with the symmetries but suppressed at low energy; this is a statement about the mathematical classification of local gauge-invariant operators by mass dimension, not an imported physical postulate.) ■

[Epistemic status: DERIVED. The action principle follows from the path sum (AP09) via the standard time-slicing representation (Feynman); the functional integral is the EH pushforward of the discrete time-sliced sum, not a reintroduction of continuum microstructure. The uniqueness of the Maxwell action uses derived symmetries (U(1), Lorentz, locality, D = 4) and the mathematical classification of gauge-invariant operators by mass dimension. No external physical postulate is imported; standard mathematical tools (variational calculus, operator classification) are applied on the derived manifold.]

Current conservation: ∂μJμ = 0. This follows from gauge invariance of the action via Noether's theorem: the action principle is derived from the path sum (Theorem 3), and U(1) symmetry gives a conserved current by Noether's first theorem; that current is Jμ. Separately, Axiom R gives ∇μTμν = 0 for the stress-

energy tensor (AP08); the two conservation laws are consistent but logically distinct.

The Bianchi identity (a geometric identity on any 2-form):

$$\partial\lambda F\mu\nu + \partial\mu F\nu\lambda + \partial\nu F\lambda\mu = 0$$

This is half of Maxwell's equations. It is automatic — it follows from Fμν being the curl of Aμ. It gives Faraday's law and the absence of magnetic monopoles. The other half — Gauss's law and Ampère's law — comes from the coupling of the connection to its source ε. The simplest action consistent with U(1) gauge invariance and Axiom R (conservation) on the 4D manifold (AP10) is:

$$S = -(1/4) \int F\mu\nu F\hat{}\{\mu\nu\}\, d^4x + \int A\mu J\hat{}\mu\, d^4x$$

Variation with respect to Aμ gives:

$$\partial\nu F\hat{}\{\mu\nu\} = J\hat{}\mu$$

These are Maxwell's equations in covariant form. You just watched them emerge from four axioms and a uniqueness argument. [Derived: manifold existence (AP20) and action principle (Theorem 3, from path sum). Given these, the field equations are unique at leading derivative order.]

The argument parallels Lovelock for gravity: given the symmetries (U(1) gauge invariance), the dimensionality (D = 4, AP10), and the conservation law (Axiom R), the action is unique at lowest derivative order, up to a coupling constant. (Higher-order gauge-invariant terms, e.g. $(F\mu\nu F\hat{}\{\mu\nu\})^2$, are consistent with the symmetries but suppressed at low energy by powers of E/Λ_UV; their absence at leading order follows from the mathematical classification of local gauge-invariant operators by mass dimension, not from an imported physical postulate.) The coupling constant is the electric charge. Its dimensionless strength is α_em.

Corollary (non-trivial curvature). In any region containing ε, Fμν ≠ 0.

Proof. *Maxwell's equations (above) give $\partial\nu F\hat{}\{\mu\nu\} = J\hat{}\mu$. The current $J\hat{}\mu \neq 0$ wherever ε is present (Theorem 1; Definition, section 4.3). Therefore $\partial\nu F\hat{}\{\mu\nu\} \neq 0$, which requires $F\mu\nu \neq 0$. This completes Theorem 2(b).* ■

5 — The Non-actualised

There is a subtlety that distinguishes the axioms' derivation from the standard gauge argument. It concerns what happens to possibilities that are not actualised.

In the pre-state, all possibilities coexist. Superposition. The empty set state. The phase is coherent across all of them because there is no separation. When a record is written (Axiom R), one possibility is actualised. The others are not. The non-actualised possibilities do not collapse by receiving a signal. No information travels to them saying "you are now impossible." They become unreachable. They were possibilities in this cycle; now they are not. Nothing propagates. Nothing violates C. The non-actualised simply cease to be available.

This is why the phase coherence of the pre-state does not require faster-than-light coordination. The phase was never coordinated across space. The phase was one, before space existed. When space emerges (R + C + G), the phase is read locally. But the reading does not require synchronisation, because the underlying reality — the pre-state — is still one.

The non-actualised possibilities become impossible in this cycle. Across infinite loops, everything with nonzero probability actualises eventually. But within this cycle, the phase coherence holds because the pre-state holds. And the connection Aμ is how the manifold expresses that holding.

5.1 — The partition

The pre-state has one resource: phase coherence. Before records, it is all one thing — maximal coherence, no manifold, no "here" vs "there." The pre-state is pure: $S(|\Psi\rangle) = 0$. All information is in correlations.

When records are written and the manifold M emerges (AP20), this phase coherence does not vanish. It is partitioned.

Every bit of the pre-state's phase coherence is either (a) written into the manifold as connection geometry, or (b) not yet written and still present in $\mathcal{H}$ as quantum correlations between subsystems. These are exhaustive categories. There is no third option.

The first account is the connection. Aμ encodes the phase coherence that has been localised — written into the manifold as curvature and holonomy. This is the pre-state's unity read as a field. It is the world of records, of particles, of history. It is the electromagnetic field. Every photon that has ever hit your eye came from this account.

The second account is entanglement. Two subsystems are entangled because they share pre-state phase coherence that has not yet been broken by a record. The EPR correlations are not signals. They are the portion of the pre-state's unity that has not yet been written into M — the possibilities that remain open, the wave side of the actualization state. Entanglement is the world of waves, of possibilities, of the quantum.

These are not two things that happen to be related. They are one thing — the pre-state's undivided phase — and the only difference is whether you are reading it as a field (connection) or as a correlation (entanglement).

5.2 — The transfer mechanism: ε

The partition is not static. Records are being written. Each record transfers phase coherence from the entanglement account to the field account. The transfer agent is ε.

Before an ε-record-event: two subsystems A and B share pre-state phase coherence. They are entangled. Between them, the connection is trivial (Aμ pure gauge, $F_{\mu\nu} = 0$). ε couples. A record is written. "Now" happens.

After the ε-record-event: the phase coherence that was shared between A and B as entanglement is now written into M as connection curvature. $F_{\mu\nu} \neq 0$ in the region (Corollary, section 4.4). The entanglement between A and B has decreased by exactly the amount of holonomy gained. The "collapse" of entanglement is not a mysterious process. It is more records being written.

This is one event, read three ways. From the field side: ε is charged (Theorem 1), it couples to the connection, $F_{\mu\nu} \neq 0$ (Corollary). From the correlation side: ε writes a record (Axiom R), which decoheres subsystems (AP13), reducing entanglement. From the thermodynamic side: every record-writing event pays a minimum cost of $k_B T \ln 2$ per bit (Landauer bound, AP06 section 4). The debit, the credit, and the receipt are three descriptions of one condition (AP06).

5.3 — The theorem

Theorem 4 (Phase Coherence Partition). The pre-state's phase coherence partitions exhaustively, under the EH pushforward, into connection curvature on M and entanglement in $\mathcal{H}$. Each ε-record-event transfers coherence from entanglement to field. The transfer is irreversible and bounded by Axiom C.

Proof. *(i) The pre-state $|\Psi\rangle$ is pure (AP09 section 3: the empty set state, superposition, the 1:1 before records). A pure state has von Neumann entropy $S(|\Psi\rangle) = 0$. All information is in correlations between subsystems, not in the state of any single subsystem.*

(ii) Under the EH pushforward (AP20), the pre-state's phase coherence is represented on M. That which has been written into records appears as the geometry of the U(1) connection — the holonomy functional, the curvature $F\mu\nu$, the field account (section 3, section 4). That which has not been written remains in $\mathcal{H}$ as quantum correlations between subsystems — the mutual information I(A:B), the entanglement account. These categories are exhaustive: every element of the pre-state's phase structure is either localised on M (a record exists) or not (no record yet). There is no third account.

(iii) The transfer mechanism is ε. ε is the only element outside Dom(σ) (Axiom B). When ε couples — when a record is written — three things happen simultaneously: (a) the connection acquires non-trivial curvature in the region (Theorem 1 → Theorem 2 → Corollary: ε is charged, forces $F\mu\nu \neq 0$); (b) the subsystems that shared phase coherence are decohered (AP13: record-writing produces decoherence); (c) a thermodynamic cost of at minimum $k_B T \ln 2$ per bit is paid (AP06 section 4: the Landauer bound applies to every record-writing event without exception). These are not three consequences of one event. They are three readings of one event (AP06).

(iv) The transfer is irreversible (Axiom R: no inverses in the record monoid). Entropy increases. The Landauer receipt cannot be recovered. Time's arrow is the direction of this transfer — from entanglement to field, from wave to particle, from possibility to record. Every measurement you have ever made paid this cost.

(v) The rate of transfer is bounded by Axiom C (Constraint). c is the speed at which records can be written — the rate at which possibilities become unreachable. This is why c appears on both sides: in the Hawking temperature

$T_H = \hbar c^3/(8\pi k_B GM)$ on the field side, in the decoherence rate on the correlation side, and in the fine structure constant $\alpha = e^2/(4\pi\varepsilon_0\hbar c)$ that governs the coupling strength. Same speed limit. Same ε. Same c. ■ (Scope note: what is proven above is the exhaustive bivalent partition under EH pushforward (i–ii), the role of ε as sole transfer mechanism (iii), irreversibility (iv), and the rate-bound (v). What is NOT proven — and is held open as KS-31 LIVE — STRUCTURAL — is the conserved-balance theorem: exhibiting a single quantity whose connection-curvature component and entanglement-entropy component are commensurable and sum to a constant under ε-record-event dynamics. The ledger language of section 5.4 expresses this conservation claim as a structural reading; the formal theorem is held open.)

5.4 — What this means

Entanglement (AP09 section 5) is not a separate phenomenon that happens to be related to the connection. It is the connection's complement — the other side of the ledger. Two particles are entangled because they share pre-state phase coherence that has not yet been written into M. The EPR correlations are not signals. They are the portion of the pre-state's unity that remains in the wave account — possibilities still open, coherence not yet localised.

Gap D asked: prove that the connection and entanglement are two faces of the same coin. The answer is: they are two columns in one ledger. The pre-state's total phase coherence is the ledger. Connection curvature (field) is the debit column — what has been written. Entanglement (correlation) is the credit column — what has not.

You cannot spend from both columns simultaneously — that is the uncertainty principle (AP12). You cannot move credits back to debits — that is Axiom R. ε is the only transfer mechanism (proven). The Landauer bound (AP06 section 4) is the receipt (proven). The columns balance because the pre-state is pure — this is the structural reading the ledger framing asserts; the formal conserved-balance theorem (a single quantity with connection-curvature and entanglement-entropy components shown to be commensurable and summing to a constant) is held open as KS-31 LIVE — STRUCTURAL. This is not a bridge between two separate structures; it is a structural reading that two names point at one object. The conservation claim the ledger language carries is the path-to-closure for KS-31.

The connection and entanglement are two readings of the same fact: the pre-state is one.

[Epistemic status: DERIVED for what Theorem 4 states (exhaustive bivalent partition, ε as sole transfer mechanism, irreversibility, rate-bound); STRUCTURAL for the conservation claim the ledger framing implicitly carries (KS-31 LIVE). Theorem 4 uses: the pre-state is pure (AP09 section 3), EH pushforward (AP20), ε is charged and forces non-trivial connection (Theorems 1–2, Corollary), ε-record-events produce decoherence (AP13), the Landauer bound applies to every record-writing event (AP06 section 4), and the record monoid is irreversible (Axiom R). All premises for the DERIVED content are derived or established. The partition is exhaustive by construction: every element of the pre-state's phase structure is either localised on M or not. No external postulate is imported. Gap D: structurally addressed by Theorem 4; formal conserved-balance theorem held open (KS-31 LIVE — STRUCTURAL).]

6 — Charge

You know what charge is now. It is the coupling of the break to the connection. But there is something remarkable about its structure — and it falls out of a theorem about circles.

6.1 — What charge is

Electric charge is the coupling of ε to the connection. ε is the break — the one element that has no σ-image. It breaks the 1:1. The connection Aμ expresses the pre-state's global phase on the manifold. Charge is how strongly the break disturbs the phase reading.

The electron carries charge −e. This is ε itself. The positron carries charge +e. This is ε read from the other sector — σ(ε) does not exist in the algebra, but the antiparticle is the conjugate state: ψ* where ψ describes ε. The conjugation is σ (Axiom S). Light and Dark. The charge flips because the sector flips.

Charge conservation is a consequence of U(1) gauge invariance (Noether's theorem). The global phase symmetry gives a conserved current. That current is the electric current. Charge conservation is the Born rule's symmetry ($\psi\psi^*$ is invariant under phase rotation) read as a conservation law on the manifold.

6.2 — Why charge is quantised

Charge comes in discrete units: ±e, ±⅓e, ±⅔e (quarks). Never ½√e or 0.7e. Always rational multiples of e.

In standard U(1) gauge theory, charge quantisation is not automatic. The Lie algebra $u(1) \cong \mathbb{R}$ admits continuous representations — any real number is a valid charge. To force discrete charges, standard physics requires extra machinery: Dirac's magnetic monopole argument, or embedding U(1) inside a compact non-abelian group (grand unification). The axioms require neither.

Theorem 5 (Charge Quantisation). All U(1) charges in the architecture are integer multiples of a minimum unit. The minimum unit is the charge of ε.

Proof. *(i) The phase group is U(1) = S^1, the circle (section 1, KS-28 CLOSED). The Born rule gives $|e^{i\theta}\psi|^2 = |\psi|^2$. The phase θ is an angle: θ and $\theta + 2\pi$ are the same phase. The symmetry group is compact.*

(ii) The irreducible unitary representations of a compact group are discrete (Peter–Weyl theorem; standard mathematics). For U(1) specifically: the irreducible representations are labelled by integers $n \in \mathbb{Z}$, each sending $e^{i\theta} \mapsto e^{in\theta}$. There is no representation labelled by $\sqrt{2}$ or π or 0.7. Only integers. This is not a physical postulate. It is a mathematical fact about circles.

(iii) The bundle structure group is U(1), not its universal cover $\mathbb{R}$ (section 3: the fiber at each point is S^1, the phase circle inherited from $\mathcal{H}$). Matter fields coupled to this bundle must transform in genuine representations of U(1), not representations of $\mathbb{R}$. The charge of any field is therefore an integer: $q \in \mathbb{Z}$. This is where the architecture's construction differs from the standard Lie-algebraic approach: section 3 constructs the bundle from the group (the circle), not from the algebra (the real line). The group is compact. Discrete representations. Integer charges.

(iv) ε carries the minimum non-zero charge. Axiom B: ε has valuation $\nu(\varepsilon) = 1$. It is the minimum element. The minimum non-zero integer is 1. Therefore ε carries charge $|q| = 1$ in units of the fundamental charge.

(v) All other charged elements in the record algebra are compositions of ε-elements. Their charges are sums of ± 1 contributions. Sums of integers are integers. Therefore all charges are integer multiples of ε's charge. ■

[Epistemic status: DERIVED USING STANDARD MATHEMATICS. Step (i) is derived (section 1). Step (ii) is the Peter–Weyl theorem (standard mathematics). Step (iii) follows from the bundle construction in section 3. Step (iv) uses Axiom B. Step (v) uses the compositional structure of the record algebra. No Dirac monopoles, no grand unification, no external mechanism is imported. The quantisation follows from: phase is a circle (derived) + circles have integer winding numbers (standard mathematics) + ε is one brick (Axiom B). Gap E: CLOSED.]

[Scope: Theorem 5 proves that all observable U(1)_EM charges are integer multiples of e. Quarks carry fractional charges ($\pm\frac{1}{3}e$, $\pm\frac{2}{3}e$), but quarks are confined — they never appear as free charges, and all hadrons carry integer charge. The fractional charges arise from the SU(3) colour structure, which is

beyond AP15's scope. AP15 derives U(1)_EM. Within U(1)_EM, observable charges are integers. The quark substructure belongs to AP19.]

7 — The Photon

The photon is the quantum of the connection. You have seen the connection derived. Now meet its minimum excitation.

The connection Aμ is a field on the manifold. When quantised (AP09 gives the quantum sector), it has excitations. The minimum excitation is one quantum: the photon.

The photon is a boson. You knew this. Now you know why. AP11 derives: paired elements (σ-image exists) are bosons, integer spin. The connection Aμ is a property of the pre-state, which has both sectors $\mathcal{C}$ and $\mathcal{P}$ (Axiom S). The phase is symmetric under σ (that is the Born rule: P = ψψ*). The quantum of a σ-symmetric field is a paired element. Paired elements are bosons. The photon is a boson.

The photon has spin 1. The connection Aμ is a 1-form: one Lorentz index. A 1-form on a 4D Lorentzian manifold transforms as a vector under the Lorentz group. The massless vector representation has spin 1. [This step uses Lorentz representation theory, which is standard mathematics on the derived manifold.]

The photon is massless. The phase symmetry is exact — U(1) is not broken. A gauge-invariant mass term $m^2 A_\mu A_\mu$ is forbidden by U(1) gauge invariance (this is a mathematical identity, not an assumption). If U(1) were broken, the photon would acquire mass via the Higgs mechanism; but U(1) is the phase freedom of the Born rule, which is exact (KS-5 CLOSED). The photon is massless because the Born rule is exact. If you could break the Born rule, you could give the photon mass. You cannot.

The photon propagates at c. Axiom C gives a finite causal bound. Maxwell’s equations (section 4.4, derived via Theorem 3) give a wave equation whose characteristic speed is c. A massless excitation of a field satisfying Lorentz-covariant dynamics propagates at the causal bound. This is the identification made in the corpus: c = finite causal bound (Axiom C). Light speed is not a property of light. It is a property of the connection. It is Axiom C read through the derived dynamics.

8 — Summary of Derivation

Axiom S → two sectors, involution σ = complex conjugation → Born rule P = ψψ* → **phase rotation e^{iθ} leaves |ψ|² invariant → U(1) symmetry.**

Axiom R + Axiom C + G → records written, manifold emerges with locality (AP20) → **the manifold is local, the pre-state is global.**

Global phase on local manifold → connection Aμ encodes phase coherence across separated points → **U(1) gauge field.** [DERIVED — ε charged (Thm 1) → non-trivial bundle (Thm 2)]

Curvature of connection → Fμν = ∂μAν − ∂νAμ → **electromagnetic field tensor.**

Axiom B → ε = electron = source of connection → **charge.**

Discrete break → quantised charge. [DERIVED — Theorem 5: U(1) compact → integer representations (Peter–Weyl) → ε = minimum (Axiom B) → all charges integer multiples of e. Gap E: CLOSED]

Axiom R (conservation) + U(1) + D = 4 (AP10) → Maxwell action unique up to coupling constant → **Maxwell's equations.**

Born rule exact (KS-5 CLOSED) → U(1) unbroken → **photon massless, spin 1, propagates at c.**

Pre-state pure (S = 0) → phase coherence partitions into field (connection curvature on M) + entanglement (correlations in $\mathcal{H}$) → ε-record-events transfer coherence from entanglement to field, irreversibly (Axiom R), rate-bounded (Axiom C), receipt k_B T ln 2 (AP06). [DERIVED for partition + transfer + irreversibility + rate-bound; STRUCTURAL for the conservation reading (KS-31 LIVE)]

9 — Kill Switches

Four kill switches engage in AP15. One existing corpus kill switch (KS-4) is affected. Each is testable. Each has a specified recovery position — if the kill switch fires, the paper reverts to a named earlier position rather than collapsing entirely. Identifiers follow the Master Kill Switch Registry.

KS-28 — Phase-group uniqueness (U(1))

Claim. The argument derives U(1) as the gauge group from the phase freedom of $\mathcal{H}$. No larger group acts on the phase of a single complex amplitude.

Test. Demonstrate that the complex Hilbert space admits a gauge symmetry larger than U(1) from the phase alone (before considering spin or colour). Failure: the derivation is overconstrained.

Status. CLOSED — MATHEMATICAL. The symmetry of a circle is U(1).

Recovery. CLOSED. No recovery needed: the symmetry group of a circle is U(1) as a mathematical fact.

KS-29 — Maxwell uniqueness

Claim. The Maxwell action S = −(1/4) ∫ FμνF^{μν} is the unique dimension-4 gauge-invariant local action (up to total derivatives) consistent with U(1) + conservation + D = 4, with the action principle derived from the path sum (Theorem 3).

Test. Demonstrate another dimension-4 gauge-invariant local action with these properties that produces different field equations. Failure: the derivation is ambiguous.

Status. CLOSED — MATHEMATICAL. Standard classification of gauge-invariant operators by mass dimension.

Recovery. CLOSED. The Maxwell action is uniquely selected at leading derivative order by the mathematical classification.

KS-30 — Phase globality

Claim. The derivation assumes the pre-state's phase is globally coherent. The pre-state is the 1:1 — it has no internal structure across which the phase could vary.

Test. Demonstrate that the pre-state can have phase discontinuities — that the white space can be internally divided. Failure: the connection is not forced to exist by the global-phase argument.

Status. LIVE — STRUCTURAL. The pre-state is the 1:1. Division of the pre-state would be a second break, violating Axiom B (one element, minimum).

Recovery. If KS-30 fires, the global-phase-on-local-manifold argument of sections 2–3 falls. The connection would have to be derived by an alternative route (perhaps by the standard gauge promotion argument). Theorems 1, 5 remain (they do not depend on global phase coherence). Theorem 4 (Phase Coherence Partition) falls because its premise (pre-state pure, S = 0) requires global coherence.

KS-31 — Entanglement–connection identification

Claim. Section 5 identifies the gauge connection and entanglement as two expressions of pre-state unity. They are not two separate structures; they are two columns in one ledger. The structural reading is: the purity of the pre-state (S = 0) entails a conservation relation between the two columns. The formal conserved-balance theorem — exhibiting a single quantity whose connection-curvature and entanglement-entropy components are commensurable and sum to a constant — is held open as the path to closure for this claim.

Test. Theorem 4 (Phase Coherence Partition) proves the identification: the pre-state's phase coherence partitions exhaustively into connection curvature (field) and entanglement (correlation), with ε as the sole transfer mechanism. Failure: identify a phase-coherent structure that is neither connection curvature nor entanglement.

Status. LIVE — STRUCTURAL. Theorem 4 (Phase Coherence Partition) establishes the exhaustive bivalent partition and identifies ε as sole transfer mechanism, but does not exhibit a conserved quantity with connection-curvature (a field action) and entanglement (a von Neumann entropy) shown to be commensurable and to sum to a constant. The partition is a structural reading of strong correspondence; the conserved-balance theorem the ledger

language asserts is not exhibited. Gap D: structurally addressed, formal balance theorem held open.

Recovery. If KS-31 fires — if a phase-coherent structure can be exhibited that is neither connection curvature nor entanglement, or if connection-curvature and entanglement-entropy cannot be made commensurable in a conserved-balance form — the structural reading remains (the partition is exhaustive by Axiom R's bivalence; ε is the transfer mechanism between field and correlation accounts), but the ledger framing is downgraded from a balance theorem to a structural metaphor. The derivations of Theorems 1, 2, 3, 5 are independent of Theorem 4 and survive. The path to closure: exhibit a single quantity — connection-curvature action plus an entanglement-entropy measure on the same footing — that is conserved under the dynamics induced by ε-record events.

KS-4 — α ≈ 1/137 (existing corpus kill switch, affected)

Claim. AP15 identifies the fine-structure constant α_em ≈ 1/137 as the coupling strength of ε to the connection (section 4.3). AP15 derives that this coupling exists; it does not derive the numerical value.

Test. Derive the numerical value of α_em from {S, B, R, C} alone. Failure of the broader corpus to do so leaves KS-4 LIVE.

Status. LIVE — HARD. Carried forward to AP24 (the six-faces formalisation of ε's multi-projection structure and the leakage-tolerance reframe on α) and AP28 (channel-counting derivation of G).

Recovery. If KS-4 is closed by AP24/AP28, no recovery needed (the result becomes derived). If KS-4 remains permanently LIVE, AP15's derivation of the structure of electromagnetism stands; only the numerical coupling strength remains empirical input. The structural results of AP15 are independent of α_em's value.

Why these four and not more

The four kill switches cover the load-bearing claims of AP15. KS-28 and KS-29 close mathematically (the symmetry of a circle is U(1); the Maxwell action is uniquely selected). KS-30 carries the structural assumption that the pre-state is undivided; this is the one remaining live falsification surface for the global-

phase argument. KS-31 is held LIVE — STRUCTURAL on Theorem 4 (the partition is a structural reading; the conserved-balance theorem the ledger framing asserts is held open). KS-4 is the existing corpus kill switch affected by AP15's identification of α_em as the coupling strength. Smaller claims throughout the paper — the specific Lovelock-type uniqueness language, the doorknob illustration, the "debit/credit/receipt" ledger metaphor, the "dictionary" framing of the connection — are illustrative rather than load-bearing. If any single illustration turned out to be poorly chosen, the paper would still stand. The kill switches engage the load-bearing structural claims, not the illustrative material.

10 — What This Establishes and What Remains Open

10.1 — What this paper closes

AP15 closes the derivation of classical electromagnetism from the axioms. Within the scope of AP15, the following are derived: U(1) gauge symmetry (from the Born rule's phase freedom); the connection Aμ on M as a U(1) principal bundle (from the global phase being read on a local manifold); gauge invariance (as standard bundle mathematics, not a postulate); the electromagnetic field tensor $F\mu\nu = \partial\mu A\nu - \partial\nu A\mu$; Theorems 1–5 with their proofs; Maxwell's equations as the unique gauge-invariant local action at leading derivative order in D = 4; the Phase Coherence Partition (Theorem 4) closing the identification of connection and entanglement as two readings of one structural fact; charge quantisation in integer multiples of ε's charge (Theorem 5); the photon's three core properties (boson, massless, spin 1, propagates at c).

Two previously open Gaps close in AP15: Gap B (minimal coupling) by Theorem 2(a); Gap E (charge quantisation formal derivation) by Theorem 5. Gap D (entanglement–connection identification) is structurally addressed by Theorem 4 — the partition is exhaustive and ε is the sole transfer mechanism — but the conserved-balance theorem the ledger framing asserts is held open (KS-31 LIVE — STRUCTURAL). Gap C (the Embedding Hypothesis) was closed earlier by AP20. Gap F (bundle construction and action principle) closes via Theorems 1–3 and the Corollary.

10.2 — What this paper does not close

AP15 does not close Gap A: the numerical value $\alpha_{em} \approx 1/137$. The derivation establishes that ε couples to the connection with some strength; the value of that strength is the work of AP24 (Residual) and AP28 (Constant). KS-4 remains LIVE.

AP15 does not close the gauge structures beyond $U(1)_{EM}$. $SU(2)_L$ (electroweak), SU(3) (colour), and the broader internal-state-space treatment live in AP16, AP19, and AP27 respectively. AP15 derives the structural template

(a global symmetry of the pre-state read through the local manifold); the specific gauge groups of the other sectors require their own derivations.

AP15 does not address the strong CP problem, the hierarchy problem, the matter-antimatter asymmetry, or the cosmological constant. These live elsewhere in the corpus or remain corpus-wide open problems.

10.3 — Items held open without claiming debt status

Whether elements of the record algebra other than ε can act as transfer agents between entanglement and field under specific structural conditions (for example, in non-abelian gauge sectors) is held open. The expectation from Axiom B (ε is the unique unpaired element) is that no other transfer agent exists at the fundamental level; the formal proof of uniqueness across all gauge sectors is not advanced in AP15.

Whether the bundle-construction logic of section 3 — a global symmetry on the pre-state read through the local manifold — extends to the non-abelian gauge groups SU(2) and SU(3) at the level of bundle topology (rather than just at the level of group structure) is held open. The expectation is that it does extend; the formal treatment lives in AP16, AP19, AP27.

10.4 — Where the philosophical-register implementations live

AP15’s structural readings have philosophical-register companions in the Ø Models catalogue.

The Phase Coherence Partition (Theorem 4) connects to Dissolutions Chapter 12 (Measurement Problem). The corpus reading: “measurement” is the writing of a record, which transfers phase coherence from the entanglement column to the field column. There is no special “measurement event” separate from record-writing. Every record-writing event is a measurement; every measurement is an ε-record-event. The “collapse” of the wave function is the partition function at work.

Maxwell’s equations as the unique gauge-invariant local action (Theorem 3 and section 4.4) connect to Resolutions Chapter 4 (Laws of Nature). The corpus reading: laws of nature are the unique configurations that satisfy the four

conditions {S, B, R, C} at a given resolution. Maxwell's equations are not imposed; they are what the four conditions plus the derived dimensionality and symmetry constraints uniquely allow.

10.5 — Structural relationships to subsequent work

AP27 (Harmonics) develops the internal-state-space and gauge-group structure that AP15's U(1) treatment introduces. The chiral coupling structure that AP27 closes (KS-63) is part of the broader treatment AP15 opens with U(1)_EM.

AP16 (Electroweak Break) breaks U(1)×SU(2) at the electroweak scale. AP15's U(1)_EM is the surviving symmetry after the electroweak break; AP16 derives the break mechanism.

AP19 (Direction) derives SU(3) colour from the gauge freedom of the break direction. AP15's structural template (global symmetry of the pre-state read through the local manifold) is the pattern AP19 extends to SU(3).

AP24 (Residual) formalises the multi-projection structure of ε that determines coupling constants. KS-4 ($\alpha_{em} \approx 1/137$) passes from AP15 to AP24 for treatment via the six-faces structure.

AP28 (Constant) derives G via channel-counting using $\varepsilon = \alpha_{em}$ from the chemical-biological boundary AP06 grounds. AP15's identification of α_{em} as the coupling strength of ε to the connection is what AP28's formula uses.

10.6 — Corpus placement

AP15 is the second chapter of Notebook IV (Forces and Constants). Its role is to derive the first of the four fundamental forces (electromagnetism) from the axioms, demonstrating the structural template that the rest of NB IV applies to the other forces. AP06 opened NB IV by grounding ε; AP15 takes the next step by deriving the simplest gauge structure that ε forces; AP27, AP16, AP19 extend the template to the more complex gauge structures; AP24, AP14, AP28 treat the constants.

10.7 — How the corpus reads now

With AP15 in place, the corpus reads as follows. Notebook I establishes the axiom and the four conditions {S, B, R, C}. Notebook II derives the constants c and G and the geometric apparatus. Notebook III derives quantum mechanics from the same axiom — the Hilbert space, the Born rule, the path sum, the uncertainty principle, the decoherence apparatus. AP06 opens Notebook IV by grounding ε in the leakage ratio. AP15 takes the next step: it derives that the U(1) phase symmetry of the Born rule, when read on the local manifold derived in AP20, produces a U(1) connection whose curvature is the electromagnetic field, with ε as the source.

Electromagnetism is not a separate force. It is the pre-state's global phase coherence, expressed on a manifold that has locality.

10.8 — The closing

The pre-state is one. The manifold is many. The connection Aμ bridges them. It is the field that says: these points appear separated, but the thing behind them — the pre-state, the white space, the 1:1 — was never divided.

The phase was always global. The manifold made it look local. The connection is what global looks like through local eyes. The electromagnetic field Fμν is the curvature of this connection. Electric and magnetic fields are the discrepancy between the global truth and the local reading. ε — the electron, the break — is the source: the point where the unity is broken and the connection must work hardest.

The photon is the minimum excitation of the connection. Massless (because U(1) is exact), spin 1 (because Aμ is a 1-form), propagating at c (because the connection carries the causal structure).

This paper derives these results from the axioms. The endpoints were already established in the corpus (U(1), the Maxwell sector, photon properties). What was missing was the middle: why ε forces the connection to be non-trivial, why the connection and entanglement are the same thing, and why charge comes in exact chunks. Theorems 1–5 close all three. ε is charged because it has no conjugate (Theorem 1). A charged source forces minimal coupling (Theorem 2a). The action principle is already inside the path sum (Theorem 3). Maxwell follows by uniqueness. $F_{\mu\nu} \neq 0$ follows from the field equations (Corollary). The

pre-state’s phase coherence partitions exhaustively into field and entanglement, with ε as the sole transfer agent (Theorem 4). Charge is quantised because the phase is a circle, circles have integer winding numbers, and ε is one brick (Theorem 5).

The derivation of classical electromagnetism from the axioms is complete. All gaps within AP15’s scope are closed. What remains open: the value of $\alpha_{em} \approx 1/137$ (Gap A / KS-4). The connection is the non-disconnection. Now you know what that means.

Separateness is experienced but not fundamental. You feel separate. The manifold makes you feel separate. Electromagnetism is the manifold’s confession of this fact.

The axiom speaks. The algebra transcribes. And now you have seen it derive electromagnetism from the phase freedom of the Born rule, the locality of the manifold, and the charge of one minimum element.

Claim Summary

Section-by-section structural enumeration with epistemic-status labels.

1 (The Phase). *DERIVATION.* U(1) is the gauge group from the Born rule's phase freedom. Not imposed; derived from Axiom S via AP09.

2 (The Pre-state is One). *DERIVATION.* The pre-state is one. The phase is global. The path sum is globally U(1)-covariant (Proposition).

3 (The Connection). *DERIVATION USING STANDARD MATHEMATICS.* A_μ is the U(1) principal bundle connection over M. The connection is the dictionary — it expresses the pre-state's global phase as read locally.

4 (Electromagnetism). *DERIVATION.* Theorem 1: ε is charged. Theorem 2: ε forces non-trivial connection and curvature. Theorem 3: action principle from path sum. Corollary: $F_{\mu\nu} \neq 0$ in regions containing ε. Maxwell's equations as the unique gauge-invariant local action at leading derivative order.

5 (The Non-actualised). *DERIVATION.* Theorem 4 (Phase Coherence Partition): the pre-state's phase coherence partitions exhaustively into connection curvature on M and entanglement in $\not{H}$, with ε as sole transfer mechanism. Gap D: structurally addressed by Theorem 4; formal conserved-balance theorem held open (KS-31 LIVE — STRUCTURAL).

6 (Charge). *DERIVATION USING STANDARD MATHEMATICS.* Theorem 5 (Charge Quantisation): all U(1)_EM charges are integer multiples of ε's charge. Compactness of U(1) + Peter–Weyl + ε is minimum (Axiom B). Gap E: CLOSED.

7 (The Photon). *DERIVATION.* Boson (AP11), massless (U(1) exact, gauge-invariant mass term forbidden), spin 1 (A_μ is a 1-form), propagates at c (Axiom C + Maxwell wave equation).

8 (Summary of Derivation). *SYNTHESIS.* Derivation chain from axioms to electromagnetism enumerated step by step.

Conditionality Footer

Dependencies

Framework: the four axioms {S, B, R, C} from Notebook I; AP05 (the structural derivation of ε); AP06 (the Landauer bound, AP06 section 4, and the leakage framework for ε ≡ η); AP08 (Einstein's field equations and the Lovelock-type uniqueness argument); AP09 (complex Hilbert space, Born rule, Schrödinger equation, path sum, σ = complex conjugation); AP10 (D = 4); AP11 (fermion/boson distinction, spin); AP12 (uncertainty principle, $\hbar$); AP13 (decoherence, environment); AP14 (time-slicing of the path sum); The Lock (Edition 04; ε = electron at chemical-biological scale); AP20 (Embedding Hypothesis, Quantum–Records Algebra).

Conditional on: None. EH and QRA proved in AP20. All other dependencies are derived corpus results.

Dependents

AP27 (Harmonics — internal state space and gauge-group structure). AP16 (Electroweak Break — U(1)×SU(2) breaking). AP19 (Direction — SU(3) colour). AP24 (Residual — multi-projection structure of ε; KS-4 treatment). AP14 (Correction — Planck-scale corrections, one-loop amplitude finiteness). AP28 (Constant — G via channel-counting using ε = α_em).

Kill switches closed by this paper

KS-28 (Phase-group uniqueness): CLOSED — MATHEMATICAL. The symmetry of a circle is U(1).

KS-29 (Maxwell uniqueness): CLOSED — MATHEMATICAL. Standard classification of gauge-invariant operators by mass dimension.

Gaps closed: Gap B (minimal coupling, Theorem 2a); Gap E (charge quantisation formal derivation, Theorem 5); Gap F (bundle construction and action principle, Theorems 1–3 and Corollary). Gap D (entanglement–connection) is structurally addressed by Theorem 4; the formal conserved-balance theorem is held open (KS-31 LIVE — STRUCTURAL).

Kill switches that remain live

KS-30 (Phase globality): LIVE — STRUCTURAL. Targets the assumption that the pre-state’s phase is globally coherent.

KS-31 (Entanglement–connection identification): LIVE — STRUCTURAL. Theorem 4 establishes the partition by exhaustion and identifies ε as sole transfer mechanism, but the conserved-balance theorem the ledger framing asserts is held open.

KS-4 (α ≈ 1/137): LIVE — HARD. Existing corpus kill switch. AP15 identifies α_em as the coupling strength but does not derive its value. Carried forward to AP24 and AP28.

Structural debts owed

Gap A (the value of α_em). AP15 derives that ε couples to the connection with some strength. It does not derive the numerical value ≈ 1/137. The path to closure runs through AP24 and AP28. [AP15 scope: not closed; carried by KS-4]

Items held open without claiming debt status

Uniqueness of ε as transfer agent across all gauge sectors. Bundle topology extension of section 3 logic to SU(2) and SU(3) (developed in AP16, AP19, AP27).

Notation Reference

U(1) — Group of phase rotations e^{iθ} on the circle S¹. The simplest continuous symmetry.

$\mathcal{H}$ — Complex Hilbert space (AP09). Carries complex amplitudes; the Born rule reads its squared magnitudes as probabilities.

M — Lorentzian manifold (AP20). D = 4 (AP10), signature (−, +, +, +).

ψ — State vector in $\mathcal{H}$. ψ = |ψ|e^{iφ} carries magnitude and phase.

σ — Involution of Axiom S; in AP09 identified with complex conjugation on $\mathcal{H}$.

ε — The unique unpaired element of Axiom B. Source of the connection. Identified with the electron at the chemical-biological scale (The Lock).

Aμ — U(1) gauge connection on M. The 1-form that encodes the pre-state's global phase as read locally.

Fμν — Electromagnetic field tensor. Fμν = ∂μAν − ∂νAμ. The curvature of the connection.

Dμ — Covariant derivative. Dμ = ∂μ − iqAμ.

Jμ — Current. Pushforward to M of the ε-record worldline. Jμ(x) = qδ³(x − x_ε(t)) dx_ε^μ/dt.

q — U(1) charge. Integer in units of e (Theorem 5).

θ — Phase angle. Element of U(1) = S¹.

α_em — Fine-structure constant. ≈ 1/137. Coupling strength of ε to the connection.

c — Speed of light. Finite causal bound of Axiom C.

ℏ — Reduced Planck constant. Quantum cost per record (AP12).

k_B — Boltzmann constant. Appears in the Landauer bound k_B T ln 2 per erased bit.

S(|Ψ⟩) — von Neumann entropy of the pre-state. S(|Ψ⟩) = 0 (pure).

AS — Actualization State. The actualizing now — the surface from which all records are written (AP01).

EH — Embedding Hypothesis. The algebraic pre-state structure embeds into the physical manifold M (AP20).

QRA — Quantum–Records Algebra. The full algebraic structure that EH embeds (AP20).

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STUDIO 6