# The Harmonics

## Artist's Proof 27

### Forces and Constants

*The electroweak gauge group from the two-sector axiom*

# Contents

# Artist’s Note

*What this paper does — and why.*

This is Artist’s Proof 27 of The 420 Code. It is the third chapter of Notebook IV (Forces and Constants). AP15 derived U(1) electromagnetism from the Born rule’s phase symmetry. AP19 derives SU(3) colour from spatial-orientation freedom. AP27 takes the next step: it derives the remaining factor of the Standard Model’s gauge group, SU(2) × U(1)_Y, from the two-sector structure of Axiom S.

The argument is a single move. Axiom S provides two sectors $\mathcal{C}$ and $\mathcal{P}$, connected by the involution σ. The break ε is the boundary between them. When ε propagates on the manifold as a quantum field, it necessarily carries an internal state encoding its relationship to each sector. That state lives in a two-dimensional complex Hilbert space $\mathcal{H}$_EW ≅ $\mathbb{C}^2$ (Lemma 1). The choice of basis on $\mathcal{H}$_EW is unphysical — only inner products are observable — so demanding consistency under local basis changes forces a gauge connection with structure group U(2) (Lemma 2). U(2) decomposes uniquely as SU(2) × U(1)_Y (Proposition 1). The SU(2) factor is weak isospin; the U(1)_Y factor is hypercharge.

Three structural results are derived: Lemma 1 (the state space exists and is $\mathbb{C}^2$), Lemma 2 (the gauge freedom forces U(2)), and Proposition 1 (U(2) factorises as SU(2) × U(1)_Y with the standard generators of weak isospin and hypercharge). A fourth result, Proposition 2 (Chiral Coupling Selection), is physically motivated and consistent with all data but is presented as structural rather than derived: the formal coupling theorem is the paper’s principal open problem. Axiom R’s arrow of time and the CPT identification (AP22) give the structural reading; the formal step from those inputs to the doublet/singlet representation split is not yet exhibited.

Seven kill switches engage: KS-61 (state-space existence), KS-62 (gauge-group overcounting), KS-63 (chiral coupling derivation), KS-64 (SU(2)_weak ≠ SU(2)_spin) are LIVE — STRUCTURAL with KS-63 carrying the paper’s principal debt. KS-65 (hypercharge assignments), KS-66 (generations), KS-67 (AP15/AP27 EWSB consistency) are OPEN DEBT — named items the paper does not address. The Standard Model’s gauge structure SU(3) × SU(2) × U(1) is now derived across three chapters: AP15 (U(1) from phase freedom), AP19

(SU(3) from orientation freedom), AP27 (SU(2) × U(1)_Y from sector-relationship freedom). Three gauge groups from three structural freedoms of ε.

# Orientation

## Where AP27 fits

The 420 Code is organised across eight bound Notebooks. AP27 is the third chapter of Notebook IV (Forces and Constants), following AP06 (The Leakage Constant) and AP15 (The Connection). AP15 derived the first of the Standard Model's three gauge groups (U(1) from phase freedom). AP19 derives the second (SU(3) from spatial-orientation freedom). AP27 derives the third (SU(2) × U(1)_Y from the relationship of ε to the two-sector structure).

AP27 depends on Notebook I (Premise) for the axioms {S, B, R, C}; on Notebook II (Spacetime) for AP05 (The Break) and AP10 (D = 4); on Notebook III (Quantum Mechanics) for AP09 (The Break — Empty Set — the complex Hilbert space structure that the break's internal state inherits), AP11 (The Spin — spin-½ from the SU(2) cover of SO(3), needed to distinguish weak SU(2) from spin SU(2)), and AP25 (The Measure — Born rule and the inner product as physical); on AP15 (The Connection) within Notebook IV for U(1) gauge structure as the post-breaking remnant; on AP19 (The Direction) for the SU(3) construction by the same mechanism applied to spatial orientation; and on AP22 (The Ledger) for the CPT identification used in the chiral coupling argument.

AP27 feeds the rest of Notebook IV and onward. AP16 (Electroweak Break) treats the symmetry-breaking mechanism that takes U(1)_Y + SU(2)_L to U(1)_EM at the electroweak scale (KS-67 EWSB consistency lives there). The hypercharge-assignment and three-generations questions (KS-65, KS-66) remain open for later treatment in the matter sector (Notebook V).

## Reading order within AP27

Front to back. Section 1 sets the stakes: two gauge groups already derived (U(1), SU(3)); one remaining; the structural feature not yet exploited for gauge purposes is Axiom S's two-sector structure. Section 2 derives the internal state space $\mathcal{H}$_EW ≅ $\mathbb{C}^2$ (Lemma 1). Section 3 derives the gauge freedom and its structure group U(2) (Lemma 2). Section 4 decomposes U(2) into SU(2) × U(1)_Y and identifies weak isospin and hypercharge (Proposition 1). Section 5 presents the structural argument for chiral coupling selection (Proposition 2, STRUCTURAL — the paper's principal open problem). Section 6 names the deeper pattern: three gauge groups, three structural freedoms, one axiom set. Section 7 lists the kill switches. Section 8 concludes.

## How to read this paper

The paper carries two voices. The structural narrative speaks directly — the central fact (the Standard Model's three gauge groups arise from three structural freedoms of ε) is stated plainly and held visible. The formal sections speak in precision. Formal blocks open with a bold label (Lemma, Proposition, Definition, Kill Switch) and close with the box mark ■. Where the argument depends on a result derived elsewhere in the corpus, the provenance is named at the point of use.

One subsection is structurally distinct from the others. Section 5 (Chiral Coupling Selection) is tagged STRUCTURAL throughout. The argument is physically motivated and consistent with all known data, and the structural inputs (Axiom R's arrow of time, AP22's CPT identification) are derived corpus results. But the formal step from those inputs to the doublet/singlet representation split is not yet exhibited. KS-63 is LIVE — STRUCTURAL and is the paper's principal open problem. Sections 1–4 stand independently of section 5; the electroweak gauge group is derived regardless of whether the chiral coupling pattern is formally derived or structurally identified.

## A note on notation

$\mathcal{C}$ and $\mathcal{P}$ are the two sectors of Axiom S, connected by the involution $\sigma: \mathcal{C} \leftrightarrow \mathcal{P}$. ε is the break — the minimal asymmetry, the boundary between sectors, the object that writes records on the manifold (Axiom R). $\mathcal{H}$_EW is the internal electroweak state space of ε: $\mathcal{H}_{EW} \cong \mathbb{C}^2$. It is distinct from spin space (AP11), colour space (AP19), and the overall phase (AP15). $|\mathcal{C}\rangle$ and $|\mathcal{P}\rangle$ form a basis for $\mathcal{H}$_EW — abstract internal labels, not spacetime directions.

U(2) is the group of 2×2 unitary matrices, the full gauge group on $\mathcal{H}$_EW before decomposition. SU(2) is the traceless (relative-orientation) part of U(2), candidate for weak isospin with generators $T^1$, $T^2$, $T^3$ (the Pauli matrices). U(1)_Y is the determinant part of U(2), candidate for hypercharge with generator Y. U(1)_EM is the post-breaking electromagnetic remnant (AP15): $Q = T^3 + Y/2$. Chirality is a Lorentz property of spinor fields; left-chiral $\psi_L$ and right-chiral $\psi_R$ are eigenstates of $\gamma^5$ obtained via the projectors $P_L = \frac{1}{2}(1 - \gamma^5)$ and $P_R = \frac{1}{2}(1 + \gamma^5)$.

# 0 — Dependency and Scope

## 0.1 — What this paper does

AP27 derives the electroweak gauge group SU(2) × U(1)_Y from Axiom S. The argument: Axiom S provides two sectors $\mathcal{C}$ and $\mathcal{P}$; the break ε is their boundary; ε's relationship to the two sectors is an internal degree of freedom; that degree of freedom lives in $\mathbb{C}^2$; the basis on $\mathbb{C}^2$ is unphysical; demanding consistency under local basis changes forces a gauge connection with structure group U(2); U(2) factorises uniquely as SU(2) × U(1)_Y; SU(2) is weak isospin, U(1)_Y is hypercharge.

The paper closes three structural questions. Lemma 1: the internal electroweak state space $\mathcal{H}$_EW exists and is isomorphic to $\mathbb{C}^2$. Lemma 2: local basis freedom on $\mathcal{H}$_EW forces a gauge connection with structure group U(2). Proposition 1: U(2) decomposes as SU(2) × U(1)_Y with the standard generators.

The paper does not close: (i) the chiral coupling theorem (Proposition 2 is STRUCTURAL — principal debt, KS-63), (ii) exact hypercharge assignments for each fermion species (KS-65), (iii) the generation structure — why three generations exist (KS-66), (iv) the Higgs mechanism or electroweak symmetry breaking (KS-67), (v) Yukawa couplings, fermion masses, mixing matrices, and anomaly cancellation conditions. These are named items the paper acknowledges without attempting.

## 0.2 — Dependencies

AP27 depends on the following derived results.

**Notebook I (Premise).** The four axioms {S, B, R, C}. Axiom S provides the two-sector structure $\mathcal{C}/\mathcal{P}$ from which the entire derivation begins. Load-bearing throughout.

**AP05 (The Break).** The structural derivation of ε as the boundary between sectors, and the Lorentzian-manifold context. Load-bearing for the chiral coupling argument (the Lorentz structure that decomposes spinor fields into chiral half-representations).

**AP09 (The Break — Empty Set).** The complex Hilbert space structure of the pre-state. Load-bearing for Lemma 1 part (ii): the internal state space must be a complex Hilbert space because the components must support superposition, interference, and the Born rule.

**AP10 (The Dimension).** D = 4 (three spatial + one temporal). Load-bearing context for the manifold on which ε propagates.

**AP11 (The Spin).** Spin-½ from the SU(2) cover of SO(3) and the fermion/boson distinction. Load-bearing for KS-64: the derived SU(2) on $\mathcal{H}$_EW must be distinguished from the Lorentz spin SU(2) of AP11; they are separate groups acting on separate spaces.

**AP15 (The Connection).** U(1) from phase freedom. Load-bearing as the structural template (global symmetry of the pre-state read locally) and as the post-breaking remnant: the AP15 U(1) is U(1)_EM, the surviving symmetry after the electroweak break, not the U(1)_Y derived in AP27. $Q = T^3 + Y/2$ connects the two.

**AP17 (The Room).** Context for the manifold structure used in the bundle construction.

**AP19 (The Direction).** SU(3) from spatial orientation freedom. Load-bearing as the second instance of the same mechanism applied to a different structural freedom — the pattern AP27 extends to the two-sector freedom.

**AP22 (The Ledger).** The CPT identification: the involution σ acts on the manifold as CPT conjugation. Load-bearing for section 5's chiral coupling argument: CPT interchanges left- and right-chiral fields, and Axiom R's arrow of time breaks CPT as a dynamical symmetry.

**AP25 (The Measure).** The Born rule and the physicality of the inner product. Load-bearing for Lemma 2: the local basis-change group is U(2) because U(2) is exactly the group of inner-product-preserving transformations on $\mathbb{C}^2$.

## 0.3 — Axiom mapping

The four axioms map to AP27 as follows.

**Axiom S (two sectors / involution).** The structural foundation of the entire paper. Axiom S provides exactly two sectors $\mathcal{C}$ and $\mathcal{P}$, connected by the involution σ. Lemma 1 reads this two-sector structure as the source of a two-component internal state on the break ε: $\mathcal{H}$_EW ≅ $\mathbb{C}^2$.

**Axiom B (unique breaking).** ε is the unique break between sectors. The whole electroweak structure is the gauge field of ε's internal state.

**Axiom R (record monotonicity).** Records are written irreversibly. The arrow of time. Load-bearing for section 5's chiral coupling argument: Axiom R breaks CPT as a dynamical symmetry, distinguishing the two chiral projections.

**Axiom C (Constraint — locality).** Finite c. Background for the Lorentzian manifold on which the fields propagate and for the locality requirement of the gauge construction.

## 0.4 — Epistemic status per section

1 (The Missing Group): [HISTORICAL] — statement of where AP27 fits within the Standard Model's gauge structure.

2 (The Internal State Space): [DERIVED USING STRUCTURAL BRIDGE] — Lemma 1 derives $\mathcal{H}$_EW ≅ $\mathbb{C}^2$ from Axiom S and the requirement that internal degrees of freedom support superposition (AP09, AP25). The two-sector count gives the dimension; the structural step from "ε has no σ-image in $\mathcal{P}$ (Axiom B)" to "ε nonetheless carries a two-component (z_$\mathcal{C}$, z_$\mathcal{P}$) internal state encoding its relationship to each sector" is an interpretive bridge — physically motivated by AP09's Hilbert space structure and AP25's Born rule, but not a fully closed derivation. The bridge is guarded by KS-61 LIVE.

3 (Gauge Freedom): [DERIVED] — Lemma 2 derives the structure group U(2) from local basis invariance and the physicality of the inner product (AP25).

4 (The Electroweak Decomposition): [DERIVED] — Proposition 1 uses standard Lie-group factorisation: U(2) ≅ SU(2) × U(1) at the algebra level.

5 (Chiral Coupling Selection): [STRUCTURAL — PRINCIPAL DEBT] — Proposition 2 is physically motivated and consistent with all data, but the formal coupling theorem is not exhibited. KS-63 LIVE.

6 (The Deeper Point): [SYNTHESIS — NON-LOAD-BEARING] — the three-gauge-groups-three-freedoms pattern.

7 (Kill Switches): [REGISTER] — falsification surfaces and named open debts.

8 (Conclusion): [SYNTHESIS] — the Standard Model's gauge structure is derived across three chapters.

## 0.5 — Kill switch summary

Seven kill switches engage in AP27. Identifiers per the Master Kill Switch Registry (global numbering).

**KS-61 (State space existence).** LIVE — STRUCTURAL. targets the construction of $\mathcal{H}$_EW from Axiom S (Lemma 1).

**KS-62 (Gauge group overcounting).** LIVE — STRUCTURAL. targets the identification of the structure group as U(2) (Lemma 2).

**KS-63 (Chiral coupling derivation).** LIVE — STRUCTURAL (principal debt). targets the formal derivation of Proposition 2; the paper's principal open problem.

**KS-64 (SU(2)_weak ≠ SU(2)_spin conflation).** LIVE — STRUCTURAL. targets the distinctness of the derived SU(2) on $\mathcal{H}$_EW from the Lorentz spin SU(2) of AP11.

**KS-65 (Hypercharge assignments).** OPEN DEBT. named item: specific Y values for each fermion species not derived.

**KS-66 (Generations).** OPEN DEBT. named item: the three-generation structure not derived.

**KS-67 (AP15/AP27 EWSB consistency).** OPEN DEBT. named item: relationship between AP15's U(1)_EM and AP27's U(1)_Y requires the EWSB mechanism, treated in AP16.

## 0.6 — Structural debts owed

**Principal debt (KS-63 — chiral coupling derivation).** Proposition 2 is STRUCTURAL: the argument from Axiom R and the CPT identification (AP22) to the doublet/singlet representation split is physically motivated and consistent with all data, but the formal coupling theorem is not yet exhibited. The work to convert STRUCTURAL into DERIVED has three steps named in section 5: (a) define the covariant derivative D_μ on the tensor product bundle (spinor ⊗

$\mathcal{H}$_EW); (b) show the coupling to P_LΨ is non-trivial (full $\mathbb{C}^2$) while the coupling to P_RΨ vanishes (reduces to Abelian U(1)); (c) derive this from Axiom R, the Lorentz structure (AP05, AP11), and the CPT identification (AP22) without importing the Standard Model's chiral coupling pattern. Steps (a) and (b) are well-defined mathematical problems. Step (c) is the hard part. [AP27 scope: not closed]

## 0.7 — Items held open without claiming debt status

Hypercharge assignments (KS-65), generations (KS-66), and EWSB consistency between AP15 and AP27 (KS-67) are held open as OPEN DEBT — named items the paper does not address. The paper derives the electroweak gauge group; it does not derive the matter content that lives in representations of that group, nor the symmetry-breaking mechanism that takes the unbroken SU(2) × U(1)_Y to the surviving U(1)_EM. These are tasks for AP16 (electroweak break) and the matter sector (Notebook V).

The global Lie-group structure U(2) ≅ (SU(2) × U(1)) / $\mathbb{Z}_2$ is noted in section 4 but its consequences for allowed representations and hypercharge quantisation are not developed in AP27. Anomaly cancellation conditions are not treated. Both are held open.

## 0.8 — Structural relationships

**Axiom S (two sectors).** The structural ground of AP27. $\mathcal{C}$ and $\mathcal{P}$ are the two sectors; σ is the involution; ε is the boundary between them. AP27 reads the two-sector structure as the source of ε's internal electroweak state space.

**AP15 (The Connection).** AP15 derives U(1) electromagnetism from phase freedom. AP27's U(1)_Y is structurally distinct from AP15's U(1)_EM: U(1)_Y is hypercharge before electroweak symmetry breaking; U(1)_EM is the surviving symmetry after. $Q = T^3 + Y/2$ connects them.

**AP19 (The Direction).** AP19 derives SU(3) colour by the same mechanism applied to spatial-orientation freedom. AP27 is the third instance of the same mechanism: identify a structural freedom of ε; demand local invariance; force a gauge connection.

**AP22 (The Ledger).** AP22 identifies σ with CPT conjugation on the manifold. AP27's section 5 uses this identification: CPT interchanges left- and right-chiral fields, and Axiom R's arrow of time breaks CPT as a dynamical symmetry. This is the structural input to the chiral coupling argument.

**AP16 (Electroweak Break).** AP16 (subsequent NB IV chapter) treats the symmetry-breaking mechanism that takes SU(2)_L × U(1)_Y to U(1)_EM. KS-67 (EWSB consistency) lives in AP16's scope. AP27 stops at the unbroken gauge group.

# 1 — The Missing Group

The Standard Model of particle physics has gauge group SU(3) × SU(2) × U(1). The axioms have derived two of the three factors.

AP15 derives U(1) from phase freedom. The complex amplitude of ε has an overall phase; local rephasing is unphysical; demanding consistency under local rephasing forces a gauge connection with structure group U(1). This is electromagnetic gauge symmetry.

AP19 derives SU(3) from spatial orientation freedom. The break ε propagates through a 3-dimensional manifold (AP05, AP10); local reorientation of the three spatial axes is unphysical; demanding consistency under local reorientation forces a gauge connection with structure group SU(3). This is colour gauge symmetry.

One factor remains: SU(2). In the Standard Model, this is weak isospin — the internal symmetry that governs the weak nuclear force. It acts only on left-handed fermion fields, organising them into doublets. Right-handed fields are singlets: the weak force does not see them.

The pattern of the previous derivations points to the answer. Each gauge group arises from a physical freedom in how ε is described:

Phase freedom → U(1). Orientation freedom → SU(3). ??? freedom → SU(2).

What degree of freedom has not yet been gauged? The axioms provide one structural feature that has not yet been exploited for gauge purposes: Axiom S. The two-sector structure $\mathcal{C}/\mathcal{P}$.

You have heard three notes of a chord. Two are identified — U(1) and SU(3). The third is sounding but unnamed. This paper names it.

# 2 — The Internal State Space

Axiom S states that reality has two sectors, $\mathcal{C}$ and $\mathcal{P}$, connected by the involution σ: $\mathcal{C} \leftrightarrow \mathcal{P}$. The break ε is the boundary between them — the object that connects the two sectors and, through Axiom R, writes records on the manifold.

The break is not purely in $\mathcal{C}$. It is not purely in $\mathcal{P}$. It is the boundary: the thing that sits between and connects them. When ε propagates on the manifold as a quantum field, it necessarily carries information about its relationship to both sectors.

This relationship is an internal degree of freedom. It is not spatial position (governed by Axiom C). It is not spin (the Lorentz spinor structure, AP11). It is not colour (spatial orientation freedom, AP19). It is not overall phase (the U(1) of AP15). It is the one remaining structural feature: the break's relationship to the two sectors.

**Lemma 1 (Internal Electroweak State Space).** The break ε, as the boundary between sectors $\mathcal{C}$ and $\mathcal{P}$ (Axiom S), carries an internal state on the manifold encoding its relationship to each sector. The space of internal states is $\mathcal{H}$_EW ≅ $\mathbb{C}^2$.

***Proof.*** *(i) Axiom S provides exactly two sectors: $\mathcal{C}$ and $\mathcal{P}$. No more, no fewer. The break connects them. On the manifold, a propagating field arising from ε carries a state that encodes how ε relates to each sector. Since there are two sectors, the state has two components: one for ε's relationship to $\mathcal{C}$, and one for ε's relationship to $\mathcal{P}$.*

*(ii) Why is this a complex linear vector space? The components must support superposition: the pre-state is a complex Hilbert space (AP09), and every quantum degree of freedom is represented as a vector in a Hilbert space. The sector-relationship components are quantum degrees of freedom — they participate in interference, entanglement, and the Born rule (AP25). A discrete label (e.g., "$\mathcal{C}$ or $\mathcal{P}$" with no superposition) would not support interference. A real vector space would not support the complex phase structure required by AP09. Therefore the internal state space is a two-dimensional complex Hilbert space.*

*(iii) The state is a vector $(z_\mathcal{C}, z_\mathcal{P}) \in \mathbb{C}^2$. The inner product on $\mathcal{H}$_EW is physical: it determines transition amplitudes via the Born rule (AP25). The internal state space is $\mathcal{H}$_EW ≅ $\mathbb{C}^2$.*

*(iv) This space is internal: not spacetime, not spin, not colour, not phase. It is the space of the break's internal orientation relative to the two-sector structure.*

Note: why exactly two components and not more? Because Axiom S provides exactly two sectors. Three sectors would give $\mathbb{C}^3$. n sectors would give $\mathbb{C}^n$. Two sectors give $\mathbb{C}^2$. The same logic as AP19: three spatial dimensions give $\mathbb{C}^3$ for colour.

Note: states in $\mathcal{H}$_EW are not merely classical labels "$\mathcal{C}$ or $\mathcal{P}$". The break can exist in coherent superpositions of $|\mathcal{C}\rangle$ and $|\mathcal{P}\rangle$. A general state $\alpha|\mathcal{C}\rangle + \beta|\mathcal{P}\rangle$ is physically meaningful. Such superpositions exhibit interference. This is what gives rise to the full continuous U(2) gauge freedom rather than a discrete $\mathbb{Z}_2$ swap.

Note: $\mathcal{H}$_EW is distinct from all previously derived spaces. In fibre bundle language: the spinor bundle, the colour bundle, and the electroweak bundle are three distinct fibre bundles over the manifold, with structure groups SU(2)_spin, SU(3), and U(2) respectively. Their fibres are different vector spaces. Their connections are independent.

# 3 — Gauge Freedom

At each point x on the manifold, the propagating field carries an internal state in $\mathcal{H}$_EW ≅ $\mathbb{C}^2$. Is the choice of basis physical?

The two basis vectors $|\mathcal{C}\rangle$ and $|\mathcal{P}\rangle$ label the break's relationship to each sector. The involution σ maps $\mathcal{C} \leftrightarrow \mathcal{P}$, so the labelling is ambiguous: σ swaps the basis. This is a discrete ($\mathbb{Z}_2$) ambiguity. But the full gauge redundancy is larger than $\mathbb{Z}_2$.

All physical observables on $\mathcal{H}$_EW depend on inner products: transition amplitudes are $|\langle\varphi|\psi\rangle|^2$ (Born rule, AP25), and expectation values are $\langle\psi|A|\psi\rangle$. Any transformation on $\mathcal{H}$_EW that preserves all inner products is physically undetectable. The group of inner-product-preserving transformations on $\mathbb{C}^2$ is U(2). The involution σ is one element of U(2). But U(2) contains all inner-product-preserving transformations, not just the swap.

Axiom S gives the two sectors equal standing — the 1:1 — and no axiom introduces a preferred basis or subgroup of U(2) on $\mathcal{H}$_EW.

**Lemma 2 (Electroweak Gauge Freedom).** The choice of basis on $\mathcal{H}$_EW at each point of the manifold is unphysical. Demanding consistency under local basis changes forces a gauge connection with structure group U(2).

***Proof.*** *(i) A local basis change on $\mathcal{H}$_EW at point x preserves the inner product on $\mathbb{C}^2$ (the inner product is physical: Born rule, AP25). The group of such transformations is U(2). The discrete involution σ ∈ U(2) motivates the basis ambiguity; the Born rule promotes it from discrete to continuous by requiring invariance under all inner-product-preserving transformations.*

*(ii) No additional structure from the axioms reduces U(2) to a proper subgroup. The physical observables on $\mathcal{H}$_EW are exhausted by inner-product structure. No further invariant structure is introduced by the axioms.*

*(iii) If the basis choice varies from point to point — if U(x) ∈ U(2) depends on position — then partial derivatives of the field pick up extra terms. A compensating gauge connection A_μ(x) must be introduced. This is the standard construction of a non-Abelian gauge theory.*

*(iv) The logic is identical to AP15 (phase → U(1)) and AP19 (orientation → SU(3)). Here: sector-relationship freedom → local U(2) invariance → electroweak gauge connection.*

Note: U(2) has four generators (the 2×2 Hermitian matrices form a 4-dimensional real vector space). Four gauge bosons. The count matches the Standard Model's electroweak sector ($W^1$, $W^2$, $W^3$, B before symmetry breaking).

You have watched the same mechanism three times now. Identify a freedom. Demand local invariance. Force a gauge connection. Phase → U(1). Orientation → SU(3). Sector-relationship → U(2). The Standard Model's gauge structure is not arbitrary. It is the axioms' internal geometry, projected onto the manifold one freedom at a time.

# 4 — The Electroweak Decomposition

The gauge group U(2) is not simple. It decomposes into two factors.

**Proposition 1 (Electroweak Decomposition).** U(2) on $\mathcal{H}$_EW decomposes as U(2) ≅ SU(2) × U(1)_Y. SU(2) acts on the relative orientation of the two sector-components (weak isospin). U(1)_Y acts on the overall phase of the doublet (hypercharge).

***Proof.*** *Every unitary matrix U ∈ U(2) can be uniquely written as U = e^{iα} · V, where α ∈ ℝ and V ∈ SU(2) (det(V) = 1). The factor e^{iα} is the determinant of U: det(U) = e^{2iα}. For the Lie algebra: u(2) ≅ su(2) ⊕ u(1).*

*The SU(2) factor acts on the relative orientation of the two components in $\mathcal{H}$_EW. It mixes |$\mathcal{L}$⟩ and |$\mathcal{P}$⟩ while preserving their total norm and the determinant. Three generators: the Pauli matrices $\sigma_1$, $\sigma_2$, $\sigma_3$ (equivalently, the weak isospin operators $T^1$, $T^2$, $T^3$).*

*The U(1) factor is the overall phase: multiplication by e^{iα} on both components simultaneously. One generator: hypercharge Y.*

*This U(1)_Y is NOT the electromagnetic U(1)_EM of AP15. The electromagnetic charge is a combination: $Q = T^3 + Y/2$. AP15 derived the post-breaking remnant. This paper derives the pre-breaking structure from which it emerges after electroweak symmetry breaking.* ■

Note on global structure: the Lie group isomorphism is U(2) ≅ (SU(2) × U(1)) / $\mathbb{Z}_2$. This global structure constrains allowed representations and is part of hypercharge quantisation. This paper derives only the local gauge algebra. Global structure, allowed representations, and anomaly constraints are open debts.

Note on hypercharge assignments: the decomposition identifies U(1)_Y as hypercharge, but the specific hypercharge values for each fermion species are not derived here. These must come from the detailed representation content of ε's internal structure and anomaly cancellation. See KS-65.

Note on AP15 consistency: AP15's U(1) is the electromagnetic remnant after symmetry breaking. AP27's U(1)_Y is hypercharge before breaking. Their consistency requires a symmetry-breaking mechanism the axioms have not yet derived. This is part of debt (iii) in the scope statement, and is the work of AP16. See KS-67.

# 5 — Chiral Coupling Selection

*Epistemic status: STRUCTURAL (PRINCIPAL DEBT). The following argument is physically motivated, consistent with all known data, and grounded in the axioms. The coupling theorem is not formally proved. This is the paper's principal open problem.*

The electroweak gauge group SU(2) × U(1)_Y has been derived. But the Standard Model has a remarkable feature: SU(2) does not couple equally to all fermion fields. It couples to left-chiral fields as doublets and to right-chiral fields as singlets. The weak force is maximally parity-violating. Why?

**Chirality vs helicity.** Chirality is a Lorentz-representation property. A Dirac spinor $\Psi$ decomposes into left-chiral and right-chiral Weyl components via $P_L = \frac{1}{2}(1 - \gamma^5)$ and $P_R = \frac{1}{2}(1 + \gamma^5)$. These transform under independent representations of the Lorentz group: $\Psi_L \in (\frac{1}{2}, 0)$ and $\Psi_R \in (0, \frac{1}{2})$.

The axioms have one structural asymmetry not yet deployed: Axiom R's arrow of time.

**Proposition 2 (Chiral Coupling Selection).** The gauge connection on $\mathcal{H}_{EW}$ couples to left-chiral fermion fields as doublets and to right-chiral fermion fields as singlets.

***Argument.*** *The break $\varepsilon$ has a direction: it proceeds from the 1:1 symmetry (pre-state) to the manifold (record). This defines a fundamental arrow (Axiom R). On the Lorentzian manifold (AP05), this arrow selects a preferred time-orientation, which selects a preferred decomposition of the Lorentz group into its two chiral half-representations.*

*The involution $\sigma$ maps $\mathcal{L} \leftrightarrow \mathcal{P}$. By AP22, $\sigma$ acts as CPT on the manifold. CPT conjugation interchanges $P_L$ and $P_R$: it maps left-chiral fields to right-chiral fields. Axiom R breaks CPT as a dynamical symmetry — the break goes forward in time, not backward. The two chiral projections are NOT dynamically equivalent.*

*For a left-chiral field $\Psi_L$, the arrow of time and the chiral structure are anti-aligned. The two internal components $z_{\mathcal{L}}$ and $z_{\mathcal{P}}$ are dynamically independent. The field carries the full $\mathbb{C}^2$ and transforms as a doublet under SU(2).*

*For a right-chiral field $\Psi\_R$, the arrow of time and the chiral structure are aligned. The alignment locks the two sector-components together. The internal $\mathbb{C}^2$ collapses to a single effective degree of freedom. The field is a singlet under SU(2).*

**What this argument needs to become a theorem.** (a) Define $D\_\mu$ on the tensor product bundle (spinor $\otimes$ $\mathcal{H}\_EW$). (b) Show the coupling to $P\_L\Psi$ is non-trivial (full $\mathbb{C}^2$) while the coupling to $P\_R\Psi$ vanishes (reduces to Abelian U(1)). (c) Derive this from Axiom R, the Lorentz structure (AP05, AP11), and the CPT identification (AP22) without importing the Standard Model's chiral coupling. Steps (a) and (b) are well-defined mathematical problems. Step (c) is the hard part.

If the coupling theorem cannot be proved, the paper still derives SU(2) × U(1)_Y as the electroweak gauge group (Lemma 1 + Lemma 2 + Proposition 1), but the chiral coupling reverts to structural identification rather than derivation.

You have watched the axiom's arrow — the irreversibility of records — reach into the internal structure of the gauge field and break the symmetry between left and right. The weak force does not see right-handed particles because the arrow of time has locked their internal doublet into a singlet. Parity violation is not a quirk of nature. It is Axiom R, projected onto the electroweak fibre.

# 6 — The Deeper Point

*Synthesis note: non-load-bearing.*

The Standard Model has three gauge groups. The axioms derive all three. Each arises from a different structural freedom of the break:

Phase freedom (how ε's complex amplitude is described) → U(1).

Orientation freedom (how ε's spatial axes are described) → SU(3).

Sector-relationship freedom (how ε's relationship to $\mathcal{C}/\mathcal{P}$ is described) → SU(2) × U(1)_Y.

Three freedoms. Three gauge groups. One axiom set. The Standard Model's gauge structure is the shape of the axioms projected onto the manifold. The harmonics were always sounding.

# 7 — Kill Switches

Seven kill switches engage in AP27. Each is testable. Each has a specified recovery position — if the kill switch fires, the paper reverts to a named earlier position rather than collapsing entirely. Identifiers follow the Master Kill Switch Registry (global numbering).

## KS-61 — Internal state space existence

**Claim.** Lemma 1 derives the internal electroweak state space $\mathcal{H}$_EW ≅ $\mathbb{C}^2$ from Axiom S's two-sector structure.

**Test.** Show that no well-defined internal ε-state space can be constructed from Axiom S. Failure: Lemma 1 fails and AP27 collapses.

**Status.** LIVE — STRUCTURAL. The construction follows the same pattern as AP15 and AP19.

**Recovery.** If KS-61 fires, no internal state space can be constructed from Axiom S. The electroweak sector requires an alternative structural ground. Lemma 2, Proposition 1, Proposition 2 all fall (they depend on Lemma 1). AP15 (U(1)_EM) and AP19 (SU(3)) are unaffected.

## KS-62 — Gauge group overcounting

**Claim.** Lemma 2 identifies the structure group of the local basis freedom as U(2). Proposition 1 decomposes U(2) into SU(2) × U(1)_Y.

**Test.** Show that the local basis freedom generates a group other than U(2) — larger or smaller. Failure: Lemma 2 and Proposition 1 fail.

**Status.** LIVE — STRUCTURAL. The argument from inner-product preservation gives U(2) as the standard gauge-theory construction.

**Recovery.** If KS-62 fires, the gauge structure has to be re-derived. Lemma 1 (state space exists) survives. Section 5 (chiral coupling) would need rewriting if the gauge group differs.

## KS-63 — Chiral coupling derivation (principal debt)

**Claim.** Proposition 2 identifies the chiral coupling pattern — left-chiral fields as doublets, right-chiral fields as singlets — through Axiom R's arrow of time and the CPT identification (AP22). The coupling theorem is STRUCTURAL: physically motivated, consistent with all data, but not yet formally proved.

**Test.** (a) Demonstrate that the chiral coupling pattern cannot be formally derived from Axiom R, the Lorentz structure, and the CPT identification — in which case the paper derives the gauge group but not the representation content. (b) Independently: if experiment showed parity conservation in the weak interaction (contradicting Wu 1957 and all subsequent data), the chiral pattern itself would fail.

**Status.** LIVE — STRUCTURAL. Principal open problem of AP27.

**Recovery.** If KS-63 fires on path (a) — if the chiral coupling cannot be derived from the structural inputs — the paper still derives SU(2) × U(1)_Y as the electroweak gauge group. The doublet/singlet split reverts to empirical input rather than structural prediction. If KS-63 fires on path (b) — experimental parity conservation — the chiral pattern as currently identified is wrong, and a deeper rethinking is required.

## KS-64 — SU(2)_weak ≠ SU(2)_spin conflation

**Claim.** Lemma 1 explicitly constructs $\mathcal{H}$_EW as an internal space distinct from spin space. The SU(2) derived in AP27 acts on $\mathcal{H}$_EW and is the weak isospin group, separate from the Lorentz spin SU(2) of AP11.

**Test.** Demonstrate that the derived SU(2) on $\mathcal{H}$_EW is identical to the Lorentz spin SU(2) of AP11. Failure: the paper has derived the wrong group.

**Status.** LIVE — STRUCTURAL. Lemma 1 explicitly distinguishes the internal sector-relationship space from spin space.

**Recovery.** If KS-64 fires, the weak isospin group is identical to the spin group, and the standard separation of internal and spacetime symmetries (Coleman–Mandula) is violated within the corpus. A deep rework would be required.

## KS-65 — Hypercharge assignments

**Claim.** Proposition 1 identifies U(1)_Y as hypercharge but does not derive specific hypercharge values for each fermion species.

**Test.** Derive the specific Y values for each fermion species from {S, B, R, C} and the AP27 structure. Until done, KS-65 stands.

**Status.** OPEN DEBT. Named item; not addressed by AP27.

**Recovery.** The closure of KS-65 requires the detailed representation content of ε's internal structure plus anomaly cancellation. This is held for the matter sector (Notebook V) and subsequent NB IV chapters.

## KS-66 — Generations

**Claim.** The Standard Model has three generations of fermions. AP27 does not derive why.

**Test.** Derive the three-generation structure from {S, B, R, C}. Until done, KS-66 stands.

**Status.** OPEN DEBT. Named item; not addressed by AP27.

**Recovery.** Held for the matter sector (Notebook V). AP27 derives the gauge structure; the matter content that fills representations of that structure is a separate question.

## KS-67 — AP15/AP27 EWSB consistency

**Claim.** AP15's U(1)_EM and AP27's U(1)_Y are structurally distinct (post-breaking remnant vs pre-breaking hypercharge). Their relationship requires electroweak symmetry breaking: $Q = T^3 + Y/2$.

**Test.** Derive the EWSB mechanism that takes SU(2)_L × U(1)_Y to U(1)_EM from {S, B, R, C} and the AP27 structure. Until done, KS-67 stands.

**Status.** OPEN DEBT. Held for AP16 (Electroweak Break).

**Recovery.** KS-67 is structurally addressed once AP16 derives the EWSB mechanism. AP27's claim that U(1)_Y ≠ U(1)_EM is consistent with the Standard Model's structure; the structural ground for the breaking is the work of AP16.

## Why these seven and how they relate

The seven kill switches partition cleanly. KS-61 and KS-62 carry Lemmas 1 and 2 — the existence and structure of the electroweak gauge connection. If either fires, the entire derivation falls. KS-64 carries the distinctness of weak SU(2) from spin SU(2) — a corpus-internal consistency check. KS-63 carries the chiral coupling pattern — the paper's principal debt. KS-65, KS-66, KS-67 are OPEN DEBTS — named items the paper does not attempt and that belong to subsequent chapters (AP16 for KS-67, the matter sector for KS-65 and KS-66). Smaller claims throughout the paper — the "three notes of a chord" framing, the "harmonics" title, the specific Lie-algebra notation choices — are illustrative rather than load-bearing. If any illustration turned out poorly chosen, the structural results would still stand.

# 8 — What This Establishes and What Remains Open

## 8.1 — What this paper closes

AP27 closes three structural results: the existence of an internal electroweak state space $\mathcal{H}$_EW ≅ $\mathbb{C}^2$ (Lemma 1); the identification of its gauge structure group as U(2) (Lemma 2); the algebraic decomposition U(2) ≅ SU(2) × U(1)_Y with the standard generators of weak isospin and hypercharge (Proposition 1).

Combined with AP15 (U(1) from phase freedom) and AP19 (SU(3) from spatial-orientation freedom), AP27 completes the derivation of the Standard Model's gauge structure SU(3) × SU(2) × U(1) at the level of the unbroken gauge group. The structural template is identified: each gauge group arises from a structural freedom of ε promoted to local invariance.

## 8.2 — What this paper does not close

AP27 does not close KS-63: the formal chiral coupling theorem. Proposition 2 is STRUCTURAL — the argument from Axiom R and the CPT identification to the doublet/singlet split is physically motivated and consistent with all data, but the formal coupling theorem is not exhibited. This is the paper's principal open problem. (Note: an earlier corpus-summary phrasing described AP27 as "closing KS-63". The source itself, and this paper, are more careful: KS-63 is LIVE — STRUCTURAL.)

AP27 does not derive specific hypercharge values for each fermion species (KS-65), nor the three-generation structure of matter (KS-66), nor the electroweak symmetry-breaking mechanism (KS-67). These are named OPEN DEBTS, held for subsequent chapters.

AP27 does not address Yukawa couplings, fermion masses, mixing matrices (CKM, PMNS), the strong CP problem, or anomaly cancellation conditions. These are corpus-wide open problems treated, where the corpus treats them at all, in the matter sector.

## 8.3 — Items held open without claiming debt status

The global Lie-group structure $U(2) \cong (SU(2) \times U(1)) / \mathbb{Z}_2$ is noted in section 4 but not developed. Its consequences for hypercharge quantisation and allowed representations are held open.

The relationship between the electroweak gauge bundle (structure group U(2)) and the spinor bundle (structure group SU(2)_spin) is treated at the level of distinctness (KS-64). The tensor product bundle (spinor ⊗ $\mathcal{H}$_EW) on which the chiral coupling theorem would live is named in section 5 but not constructed.

## 8.4 — Where the philosophical-register implementations live

AP27's structural reading has a philosophical-register companion in the Ø Models catalogue. The pattern — three gauge groups from three structural freedoms of one break — connects to Resolutions Chapter 4 (Laws of Nature). The corpus reading: the Standard Model's gauge structure is not a list of empirical inputs but the shape of the four conditions {S, B, R, C} projected onto the manifold via the freedoms of ε. Laws of nature are the unique configurations the four conditions allow at the resolution of measurement.

## 8.5 — Structural relationships to subsequent work

AP16 (Electroweak Break) takes the unbroken SU(2)_L × U(1)_Y derived in AP27 and treats the symmetry-breaking mechanism that leaves U(1)_EM as the surviving symmetry. KS-67 EWSB consistency lives in AP16's scope.

AP24 (Residual) formalises the multi-projection structure of ε. The relationship between the projections of ε into the four faces (or six, depending on the AP24 formalisation) and the gauge representations is held open at the AP27 level; AP24 may inform the hypercharge assignments (KS-65).

The matter sector (Notebook V) treats the fermion content: the three generations, the mass spectrum, the mixing matrices. KS-66 (generations) lives in that scope.

## 8.6 — Corpus placement

AP27 is the third chapter of Notebook IV (Forces and Constants). Its role is to complete the derivation of the Standard Model's gauge structure. AP06 opened NB IV by grounding ε; AP15 derived the first gauge group (U(1)_EM); AP19 derives the second (SU(3)); AP27 derives the third (SU(2) × U(1)_Y). With AP27 in place, the gauge sector of NB IV has its structural template complete. AP16 then treats the breaking; AP24, AP14, AP28 treat the constants.

## 8.7 — How the corpus reads now

With AP27 in place, the corpus reads as follows. The Standard Model has three gauge groups: U(1), SU(2), SU(3). The 420 Code derives all three from the same structural mechanism applied to three different freedoms of the break ε. Phase freedom → U(1) (AP15). Orientation freedom → SU(3) (AP19). Sector-relationship freedom → SU(2) × U(1)_Y (AP27). Three freedoms. Three gauge groups. One axiom set.

The Standard Model's gauge structure is not arbitrary. It is the shape of {S, B, R, C} projected onto the manifold, one freedom at a time. The harmonics were always sounding.

## 8.8 — The closing

The electroweak gauge group is derived from Axiom S. The break ε carries an internal two-component state encoding its relationship to the two sectors: $\mathcal{H}$_EW ≅ $\mathbb{C}^2$ (Lemma 1). Local basis invariance forces a gauge connection with structure group U(2) (Lemma 2). The algebraic decomposition U(2) ≅ SU(2) × U(1)_Y identifies weak isospin and hypercharge (Proposition 1).

The chiral coupling pattern is structurally identified through Axiom R's arrow of time and the CPT identification (Proposition 2, structural, principal debt). KS-63 carries the remaining work — the formal coupling theorem that would convert Proposition 2 from STRUCTURAL to DERIVED.

The Standard Model's gauge structure SU(3) × SU(2) × U(1) is now derived: AP15 (U(1) from phase freedom), AP19 (SU(3) from orientation freedom), AP27 (SU(2) × U(1)_Y from sector-relationship freedom). Three gauge groups from three structural freedoms of ε. The harmonics were always sounding.

# 9 — Conclusion

The electroweak gauge group is derived from Axiom S.

The break ε carries an internal two-component state encoding its relationship to the two sectors: $\mathcal{H}_{EW} \cong \mathbb{C}^2$ (Lemma 1). Local basis invariance forces a gauge connection with structure group U(2) (Lemma 2). The algebraic decomposition $U(2) \cong SU(2) \times U(1)_Y$ identifies weak isospin and hypercharge (Proposition 1).

The chiral coupling pattern is structurally identified through Axiom R's arrow of time and the CPT identification (Proposition 2, structural, principal debt).

The Standard Model's gauge structure $SU(3) \times SU(2) \times U(1)$ is now derived: AP15 (U(1) from phase freedom), AP19 (SU(3) from orientation freedom), AP27 ($SU(2) \times U(1)_Y$ from sector-relationship freedom). Three gauge groups from three structural freedoms of ε.

# Claim Summary

Section-by-section structural enumeration with epistemic-status labels.

**1 (The Missing Group).** *HISTORICAL.* Statement of where AP27 fits within the Standard Model's gauge structure. Two factors already derived (U(1), SU(3)); one remaining; the structural feature not yet exploited is Axiom S's two-sector structure.

**2 (The Internal State Space).** *DERIVED USING STRUCTURAL BRIDGE.* Lemma 1: the break ε carries an internal state on the manifold encoding its relationship to each sector; the space of internal states is $\mathcal{H}$_EW ≅ $\mathbb{C}^2$. The two-sector count gives the dimension; the bridge from Axiom B (no σ-image) to a two-component state is interpretive and is guarded by KS-61.

**3 (Gauge Freedom).** *DERIVED.* Lemma 2: the local basis on $\mathcal{H}$_EW is unphysical; demanding consistency under local basis changes forces a gauge connection with structure group U(2).

**4 (The Electroweak Decomposition).** *DERIVED.* Proposition 1: U(2) ≅ SU(2) × U(1)_Y; SU(2) is weak isospin with generators $T^1$, $T^2$, $T^3$ (Pauli matrices); U(1)_Y is hypercharge with generator Y.

**5 (Chiral Coupling Selection).** *STRUCTURAL — PRINCIPAL DEBT.* Proposition 2: the gauge connection on $\mathcal{H}$_EW couples to left-chiral fermion fields as doublets and to right-chiral fermion fields as singlets. Physically motivated, consistent with all data; formal coupling theorem not exhibited. KS-63 LIVE.

**6 (The Deeper Point).** *SYNTHESIS — NON-LOAD-BEARING.* Three freedoms → three gauge groups → one axiom set. The Standard Model's gauge structure is the axioms' internal geometry projected onto the manifold.

**7 (Kill Switches).** *REGISTER.* KS-61 through KS-67 in Claim/Test/Status/Recovery form.

**8 (What This Establishes and What Remains Open).** *SYNTHESIS.* Closure of three structural questions; principal debt at KS-63; open debts at KS-65, KS-66, KS-67; corpus placement and forward reading.

**9 (Conclusion).** *SYNTHESIS.* The Standard Model's gauge structure SU(3) × SU(2) × U(1) is now derived across three chapters: AP15, AP19, AP27.

# Conditionality Footer

## Dependencies

Framework: the four axioms {S, B, R, C} from Notebook I; AP05 (The Break — Lorentzian manifold and ε's structural role); AP09 (The Break — Empty Set — complex Hilbert space structure); AP10 (D = 4); AP11 (The Spin — spin-½ from SU(2) cover of SO(3)); AP15 (The Connection — U(1)_EM as post-breaking remnant); AP17 (manifold context); AP19 (The Direction — SU(3) from orientation freedom); AP22 (The Ledger — CPT identification); AP25 (The Measure — Born rule and inner product physical).

Conditional on: Nothing. EH and QRA proved in AP20. All other dependencies are derived corpus results. The axioms themselves are unconditional.

## Dependents

AP16 (Electroweak Break) — takes SU(2)_L × U(1)_Y to U(1)_EM via the EWSB mechanism. AP24 (Residual) — may inform hypercharge assignments via the multi-projection structure of ε. The matter sector (Notebook V) — fills the gauge representations with fermion content; treats the three generations.

## Kill switches engaged

KS-61 (state-space existence): LIVE — STRUCTURAL. Lemma 1.

KS-62 (gauge-group overcounting): LIVE — STRUCTURAL. Lemma 2.

KS-63 (chiral coupling derivation): LIVE — STRUCTURAL (principal debt). Proposition 2.

KS-64 (SU(2)_weak ≠ SU(2)_spin): LIVE — STRUCTURAL. Distinct from AP11's spin SU(2).

KS-65 (hypercharge assignments): OPEN DEBT. Held for matter sector.

KS-66 (generations): OPEN DEBT. Held for matter sector.

KS-67 (AP15/AP27 EWSB consistency): OPEN DEBT. Held for AP16.

## Structural debts owed

Principal debt: KS-63 (chiral coupling derivation). Proposition 2 is STRUCTURAL; the formal coupling theorem is the paper's principal open problem. Path to closure: (a) construct the tensor product bundle (spinor $\otimes$ $\mathcal{H}$_EW); (b) show the coupling to P_LΨ is non-trivial while the coupling to P_RΨ vanishes; (c) derive both from Axiom R + Lorentz structure + CPT identification (AP22) without importing the Standard Model's chiral pattern. Step (c) is the hard part.

## Items held open without claiming debt status

Global Lie-group structure $U(2) \cong (SU(2) \times U(1)) / \mathbb{Z}_2$ and its implications for representations and hypercharge quantisation. Anomaly cancellation conditions. Yukawa couplings, fermion masses, mixing matrices.

# Notation Reference

**$\mathcal{C}$, $\mathcal{P}$** — The two sectors of Axiom S, connected by the involution $\sigma$: $\mathcal{C} \leftrightarrow \mathcal{P}$.

**ε** — The break. The minimal asymmetry. The boundary between sectors; the object that writes records on the manifold.

**σ** — The involution of Axiom S. In AP22 identified with CPT conjugation on the manifold.

**$\mathcal{H}$_EW** — Internal electroweak state space of ε. $\mathcal{H}_{EW} \cong \mathbb{C}^2$. Distinct from spin space (AP11), colour space (AP19), and overall phase (AP15).

**$|\mathcal{C}\rangle$, $|\mathcal{P}\rangle$** — Basis for $\mathcal{H}_{EW}$. Abstract internal labels, not spacetime directions.

**U(2)** — Group of 2×2 unitary matrices. The full gauge group on $\mathcal{H}_{EW}$ before decomposition.

**SU(2)** — Traceless (relative-orientation) part of U(2). Weak isospin.

**U(1)_Y** — Determinant part of U(2). Hypercharge.

**U(1)_EM** — Electromagnetic gauge symmetry (AP15). The post-breaking remnant: $Q = T^3 + Y/2$.

**$T^1$, $T^2$, $T^3$** — Weak isospin generators (Pauli matrices $\sigma_1$, $\sigma_2$, $\sigma_3$).

**Y** — Hypercharge generator.

**Q** — Electromagnetic charge. $Q = T^3 + Y/2$ (post-breaking).

**Chirality** — Lorentz-representation property of spinor fields. Left-chiral $\psi_L \in (\frac{1}{2}, 0)$ and right-chiral $\psi_R \in (0, \frac{1}{2})$.

**$\gamma^5$** — Chirality operator. Eigenvalues ±1 distinguish left-chiral and right-chiral components.

**P_L, P_R** — Chiral projectors. $P_L = \frac{1}{2}(1 - \gamma^5)$, $P_R = \frac{1}{2}(1 + \gamma^5)$.

**D_μ** — Covariant derivative. For the electroweak connection: $D_\mu = \partial_\mu - igW_\mu^a T^a - ig'B_\mu Y/2$.

**AS** — Actualization State. The actualizing now.

**EH** — Embedding Hypothesis (AP20). The algebraic pre-state structure embeds into the physical manifold.

**QRA** — Quantum–Records Algebra (AP20). The full algebraic structure that EH embeds.

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